Competing Principals? Legislative Representation in List PR Systems*

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Abstract

We develop a new framework to study legislative representation in list proportional representation (PR) systems. Rejecting the classical closed-versus-open dichotomy, we instead conceptualize list PR as a continuum, reflecting different degrees of *list flexibility*. We contradict the conventional wisdom that relatively closed lists strengthen party control, and thus legislative cohesion, at the expense of local representation. In particular, we show that less flexible lists can generate superior incentives for legislators to advance the interests of their local constituents. We also find that legislative cohesion under open lists may approach levels associated with closed lists, or fall strictly below levels associated with single-member contexts. A more polarized electorate increases legislative cohesion if and only if list flexibility is low enough.

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1. Introduction

About two thirds of the world's democratic legislatures are elected via list proportional representation (PR).[1] Under *closed* lists, voters can only cast a ballot in favor of a party, and seats are awarded to candidates in the order in which they appear on the ballot. Under *open* lists, voters may cast a "preference" vote for an individual candidate on a party list; seats are then awarded to the candidates that, in each party list, receive the largest shares of preference votes. The fundamental dichotomy in existing research is therefore between *closed* list systems, in which voters have no opportunity to express preferences for individual candidates, and *open* list systems, where they can influence which specific candidates represent them.

Building on this dichotomy, the prevailing scholarly wisdom is that closed list systems encourage cohesive parties, while open list systems create a stronger link between representatives and their constituents. The choice between closed and open lists should therefore have a fundamental impact on the nature of political representation: closed lists strengthen party control and legislative cohesion at the expense of local representation, while open lists strengthen local representation at the expense of legislative cohesion. This perspective is maintained in most academic literature (e.g., Carey and Shugart, 1995, Kunicova and Rose-Ackerman, 2005, Chang and Golden, 2007, André and Depauw, 2013), widely employed datasets (Scartascini, Cruz and Keefer, 2018) as well as prescriptions to policy practitioners (e.g., Reynold, Reilly and Eliis, 2005).

In this paper, we develop a new theory of legislative representation in party-list PR that departs from this dichotomy. By organizing different systems on a continuum of *list flexibility*, we show that the scholarly wisdom is incorrect: too much flexibility worsens the representation of local interests *and* reduces cohesion.

Our theory acknowledges that, in practice, many systems combine features of *both* open *and* closed lists: seats are awarded to candidates in the order they are ranked by parties, unless a candidate's share of preference votes within her party's list exceeds a pre-specified threshold. In that event, she acquires priority over her co-partisans *regardless* of her position on the list. Countries using variants of this system include Austria, Belgium, the Czech Republic, Denmark, Estonia, Indonesia, Norway, Slovakia, and Sweden. A more flexible list corresponds to a lower threshold of preference votes that a candidate needs in order to circumvent the party's list rank.

List flexibility therefore reflects the relative importance of preference votes versus a favorable rank assignment for a candidate's prospect of election. A completely flexible list—where pref-

[1] According to the *Database of Political Institutions* (Cruz, Keefer and Scartascini, 2016), as of 2015 legislative elections in 94 out of 147 democracies employ proportional representation, of which 25 feature flexible or open lists.

erence votes alone determine the order in which seats are filled—corresponds to open list PR. A completely inflexible list—where the party's ranking alone determines the order in which seats are filled—corresponds to closed list PR.

A small number of empirical papers have progressed beyond the open versus closed dichotomy; they highlight how a distinction between *closed*, *open* and *ordered* systems (Farrell and Scully, 2007), or between *weakly* versus *strongly* flexible lists (Däubler and Hix, 2017) more effectively organizes empirically observed patterns of legislative voting. In particular, they show that the traditional distinction between open and closed lists "hides that additional features may cause important variation in intra-party competition between different [...] systems" (Bräuninger, Brunner and Däubler, 2012, p. 609). By taking this criticism seriously, our framework yields novel insights into the consequences of different forms of list PR for legislative representation and legislative cohesion.

First, we show that the relationship between list flexibility and local representation is non-monotonic: there exists a critical threshold of list flexibility above which a more flexible list *weakens* representatives' incentives to advance their constituents interests. In particular, if office-holding motives weigh sufficiently on politicians, representatives are better-motivated to serve their constituents under *closed* (i.e., completely inflexible) lists than under *open* (i.e., completely flexible) list systems. These results contradict the common wisdom that more candidate-centered systems produce superior representation of local interests at the expense of cohesive legislative parties.

Second, we show that flexible systems may induce legislative behavior—in particular, voting cohesion within parties—that is observationally equivalent to behaviors arising under open and closed list systems, consistent with existing evidence (Sieberer, 2006). In a sample of nineteen countries, Sieberer finds that countries with open list variants achieved both the *lowest* (Finland) and the *highest* (Denmark) measures of legislative cohesion. In light of the traditional dichotomy, he interprets this pattern as evidence of a null effect of list type on legislative cohesion. Our analysis, by contrast, shows that these findings are fully consistent with a first-order role of electoral institutions: while Finland mandates an open (i.e., maximally flexible) list, Denmark gives parties the option to use a much less flexible list (Elklit, 2005).

Our Approach. Our framework builds upon two central ideas from the comparative politics literature. First, as elaborated in seminal work by Hix (2002) and Carey (2007), representatives are simultaneously and distinctively accountable to two *competing principals*: their constituency voters and their legislative party leaders. Second, multi-member districts create a tension between the value of *individual* and *collective* reputation (Carey and Shugart, 1995; Shugart, 2005): actions that enhance an individual legislator's reputation can either advance or harm the ticket's collective reputation.

Our model features a continuum of local voters in a single constituency, represented by two Incumbent representatives facing two Opposition candidates, and a national party leadership that is trying to advance its legislative policy agenda. Voters rely on their representatives to act in their interest by choosing when to toe the party line, and when instead to oppose it.

Relative to their representatives, voters face two key informational shortcomings. First, they cannot fully assess whether the party leadership's agenda also benefits their local interests. Second, they are concerned about their representatives' policy priorities (Crisp, Olivella, Malecki and Sher, 2013): they do not know whether each representative is *aligned*, i.e., values only policies that benefit her constituents, or *mis-aligned*, i.e., is willing to sacrifice constituency interests on the altar of her party's legislative accomplishments. Voters rely on legislators' voting records to inform their opinions about each of their representatives' alignment.

A representative's perceived alignment increases when she votes against her party leadership, and falls when she toes the party line. However, the magnitude of this reputation effect depends on how other representatives vote: supporting the party's agenda is much more damaging to her reputation when her co-partisan opposes it. The resulting incentives connect our work to formal models of pandering (e.g., Canes-Wrone, Herron and Shotts, 2001, Groseclose and McCarty, 2001, Fox and Van Weelden, 2010), as well as theories of electoral externalities in (single-member) multi-district settings (Zudenkova, 2011, Zudenkova, forthcoming, Buisseret and Prato, 2016).

In multimember districts, however, legislators compete both as a team to win as many seats as possible, as well as with one another for seats in the common event that their list fails to elect all its candidates. Their actions simultaneously affect two electoral contests: an *inter*-party contest, and an *intra*-party contest. The reason is that a preference vote has two competing effects. First, it raises the total vote share—and the expected number of seats—of the candidate's party, thereby conveying a common benefit to all politicians from the party. Second, it also raises the within-ticket share of preference votes awarded to that candidate. This improves her prospect of election at the direct expense of her co-partisan's. As a consequence, a representative's prospect of reelection may not increase when her party's electoral performance as a whole improves.

As long as the list is not completely flexible—i.e., open—seats are filled in the order that candidates are ranked unless a candidate receives a sufficiently large share of the preference votes versus her co-partisans. All else equal, a higher-ranked candidate is more likely to be elected than a lower-ranked candidate. Through their choice of list assignment, party leaders thus compete with voters in shaping the intra-party contest. Actions that please voters may be rewarded with a larger share of preference votes, while actions that please party leaders may be rewarded with a higher list ranking.

Which channel is most salient in a legislator's calculation depends on the list flexibility. A very flexible list, i.e., approaching open PR, implies that a low-ranked candidate requires a relatively small (within-list) share of preference votes within the party's ballot in order to secure reelection. A very inflexible list, i.e., approaching closed PR, implies that a low-ranked candidate requires a very large share of preference votes in order to secure reelection.

These considerations generate the fundamental trade-off that an incumbent representative faces. First, she may cultivate a "personal vote" (Carey and Shugart, 1995) by opposing her party's legislative agenda, in an attempt to convince voters of her alignment with their interests. Second, she may throw her support behind the party's agenda in order to curry favor with the party leadership and obtain a more favorable list assignment. Either strategy will change her individual reputation, but since politicians are evaluated as a team, her strategy also changes the *collective reputation* of the party's incumbent representatives, and thus the party's overall performance.

Results. Our main results focus on two key outcomes: dyadic representation (Stokes and Miller, 1962)—i.e., "how well the sitting legislator acts as an agent for the constituency on legislative decisions" (Ansolabehere and Jones, 2011, 1)—and legislative cohesion—i.e., the propensity for incumbent legislators to support their party's legislative agenda.

Flexibility and Dyadic Representation. A conventional wisdom states that, relative to closed lists, open lists should induce representatives to more effectively advance the interests of their local voters, since they increase intra-party competition for preference votes, rather than concentrating power in the hands of party leaders. Our results contradict this intuition: empowering voters by making preference votes more electorally consequential—relative to party leaders' list assignment—can actually *worsen* dyadic representation. The reason is that competition for preference votes encourages representatives to build personal reputation by voting against their parties. When this competition it too intense, it generates excessive obstruction, even encouraging incumbents to vote against policies that would benefit their voters.

In this context, a less flexible list transfers electoral control from voters to party leaders—whose induced preferences over representatives run completely contrary to voters. But precisely by weakening candidates' incentives to cultivate a personal reputation for alignment, this may ultimately *improve* dyadic representation. We identify a critical threshold of list flexibility above which a more flexible list *worsens* legislators' ability to advance the interest of their geographic constituency. Formally stated: the quality of dyadic representation is strictly quasi-concave in the degree of list flexibility. In addition, we show that if office-holding motives are not too small, a completely flexible system (i.e., open lists) leads to worse dyadic representation than a completely inflexible system (i.e., closed lists).

More broadly, our results suggest that improving legislative cohesion needs not come at the expense of a weaker geographic link between representatives and local interests. While it is commonly assumed that intra-party competition improves legislators' attentiveness to local interests (Ames, 1995; Carey and Shugart, 1995; Crisp, Escobar-Lemmon, Jones, Jones and Taylor-Robinson, 2004; Hallerberg and Marier, 2004), our framework highlights how the struggle for electoral control of representatives is not always a zero-sum game between voters and party leaders. If attempts to build personal reputation cause representatives to act in ways that do not benefit their districts *or* their parties, a transfer of control from voters to parties may benefit *both* voters *and* party leaders. Put more succinctly: *principals do not always compete*.

Legislative Cohesion and Polarization. We show that list flexibility is a crucial moderating variable for the relationship between polarization and legislative cohesion. Specifically, we unearth a critical threshold of flexibility below which an increase in district voters' partisan polarization *increases* incentives to vote with the party leadership, but above which the same change in polarization *lowers* incentives to vote with the party leadership.

To see why, notice that in a highly polarized electorate the distribution of votes *across* lists is not highly responsive to representatives' actions: voters are instead largely guided by their party preferences. As a result, more partisan polarization generates a more predictable distribution of votes—and thus of seats—across parties. Nonetheless, partisan polarization has no consequence for voters' preferences over candidates *within the same list*. From an incumbent's perspective, this heightens the relative salience of the intra-party contest as the key to her electoral fortunes.[2]

Thus, whether an increased emphasis on the intra-party competition raises or lowers legislative obstruction depends critically on the flexibility of the list. If the list is not too flexible, more intra-party competition translates into more competition for the favor of party leaders, encouraging representatives to *toe* the party line. But if the list is sufficiently flexible, more intra-party competition translates into more competition for preference votes amongst co-partisans, encouraging representatives to *oppose* the party line.

Legislative Cohesion and Ballot Structure. Consistent with conventional wisdom, our model predicts that cohesion decreases with the flexibility of the list. But our framework also shows that legislative cohesion under list PR can be *even lower* than under single-member systems. This con-

[2] Rudolph and Däubler (2016) also note that open list systems allow voters to separate their party preferences from their preferences over candidates. They argue that this can increase individual accountability. As we will subsequently show, however, our analysis cautions that excessive intra-party competition—arising in very flexible list settings—may lead incumbents to engage in personal vote-seeking that ultimately undermines representation.

tradicts the view that list PR systems should be associated with higher levels of party discipline than majoritarian systems (notably, Carey and Shugart, 1995).

To see why, notice that in a single-member context, incumbents represent distinct districts, and thus are accountable to distinct electorates. This implies that there is no direct competition within the pool of incumbents for the same seat.[3] In a multimember district, by contrast, multiple incumbents compete for the same seat, implying that one incumbent's reelection may come at the expense of another incumbent from the same party. In a sufficiently flexible system, the resulting incentive to oppose the party agenda leads to even more obstruction than in a single-member majoritarian system, as each incumbent jockeys to build a personal reputation at the others' expense. We find that this wedge in cohesion is especially heightened for unpopular incumbent parties, whose prospects of winning many seats are small and where the conflict of interest between incumbents on the same list is therefore maximized in a multi-member district.

Related Literature. Almost all existing formal theoretical studies of PR focus on national policy outcomes. Existing work studies how national governments form after multi-party elections (e.g., Austen-Smith and Banks, 1988, Baron and Diermeier, 2001), how citizens vote strategically (Cox and Shugart, 1996, Cox, 1997, Cho, 2014) and how PR shapes policy outcomes such as the size and composition of government spending (e.g., Austen-Smith, 2000, Persson and Tabellini, 2005), Lizzeri and Persico, 2001) and corruption (Persson, Tabellini and Trebbi, 2003). Yet, the defining characteristic of a PR system is not an institutional property of *national* politics, but rather of *local* politics: the election of multiple representatives in each district (multimember districts, or MMD).

Some recent work focuses more explicitly on parties' *internal* dynamics in MMD contexts, but with very different objectives and mechanisms compared to our paper: how rank assignment mechanisms can solve intra-party moral hazard problems (Crutzen, Flamand and Sahuguet, 2018) and provide voters with valuable informational cues (Patty, Schibber, Penn and Crisp, forthcoming), or how the electoral systems affects minority representation Negri (2018) and intra-party ideological dispersion (Matakos, Savolainen, Troumpounis, Tukiainen and Xefteris, 2018).

Organization of the Paper. We first present our model, and characterize the equilibrium. We then show how incentives to cultivate a personal vote vary with list flexibility and district inter-party polarization. We then compare legislative cohesion under PR versus single-member districts. We subsequently highlight our results on the consequences of greater flexibility for dyadic representation. We then show how our insights are robust to a variety of extensions and generalize to richer settings, including any district magnitude. A conclusion follows. Proofs are in the Appendix.

[3] While primary elections are a form of intra-party contest in single-member contexts, at most one incumbent competes—distinguishing these contests from our multi-member setting.

2. Model

Agents. We consider a two-date interaction between a unit mass of constituency voters, two incumbent co-partisan legislators representing the constituency (A and B, to whom we reserve the pronoun "she"), and their party leadership (L). The party leadership represents any individual or collective authority in the incumbents' party—for example, a senior legislative leadership or national party executive. At the end of the first date, the incumbents compete for reelection against two opposition politicians.[4] There are three types of voters: committed *Incumbent* supporters, committed *Opposition* supporters, and *uncommitted* voters. For simplicity, we assume that committed Incumbent supporters and committed Opposition supporters each constitute a fraction $\alpha < .5$ of the electorate. Thus, there are $1 - 2\alpha$ uncommitted voters. At each date, there is a *legislative interaction*, and between the first and second date there is an *election*.

Legislative Interaction. First, a national policy initiative comes before the floor of the legislative body—for example, a public infrastructure program, an economic reform, or an international agreement that requires domestic ratification. The local consequences of this policy for the district are initially uncertain; for example, it may be difficult for voters to fully anticipate the consequences of a new international trade agreement for their district. Specifically, the value to the district's constituents from the policy is a random variable θ, uniformly distributed on $[-\kappa, \kappa]$. The value θ is privately learned by each representative, while the party leadership and the voters do not observe it.[5] If the policy is not implemented, all agents derive a status quo policy payoff of zero.[6] Thus, $\theta > 0$ implies that the policy yields a net *benefit* to the district, while $\theta < 0$ implies that the policy yields a net *harm* to the district.

Second, each representative simultaneously supports or opposes the policy agenda by choosing either *aye* (y) or *nay* (n). A *vote tally* $\mathsf{t} = (\mathsf{t}_i, \mathsf{t}_j) \in \{\mathsf{y}, \mathsf{n}\}^2$ determines the probability $q(\mathsf{t})$ that the agenda passes. For simplicity, we impose:

$$q(\mathsf{y}, \mathsf{y}) - q(\mathsf{n}, \mathsf{y}) = q(\mathsf{y}, \mathsf{n}) - q(\mathsf{n}, \mathsf{n}) \equiv \chi > 0,$$

[4] Our benchmark model assumes that both incumbent representatives in the district are from the same party. However, this is for ease of presentation and is not required for our results.

[5] The assumption that the leadership does not observe θ is unimportant: all our results hold when the leadership learns θ. The crucial feature is that the opacity of parliamentary negotiations and the inherent uncertainty associated with policy outcomes generates substantial uncertainty on the party of voters about the local consequences of legislation.

[6] This normalization is without loss of generality: one may interpret θ as the *relative* value to the constituency from the project, i.e., relative to the status quo.

which implies that the prospect that a bill passes is increasing in the number of votes in favor.[7]

Election. After the leadership and voters observe incumbents' vote tally (t), the party leadership L constructs the party *list* assignment; this specifies the order in which the incumbents will appear on the ballot: either $\{AB\}$ or $\{BA\}$.

After the leadership constructs its list, the election takes place. Voters may vote for (i) a candidate on the Opposition ticket, or (ii) incumbent A, or (iii) incumbent B. In keeping with the terminology used in existing literature, we refer to a vote for either of A or B as a *preference vote*, and denote by v_A and v_B the number of preferences votes cast in favor of each incumbent representative. Thus, the total number of preference votes in favor of the opposition ticket is $1-v_A-v_B$. Depending on the number of votes and the list assignment, an election determines whether each incumbent representative $i \in \{A, B\}$ either keeps her seat ($e(i) = 1$), or loses it ($e(i) = 0$). We let $e = \{e(A), e(B)\}$.

Allocation of Seats Between Parties. The total vote share of the party's ticket is $v_A + v_B$ can yield zero, one, or two seats, according to the following mechanism:

1. if $v_A + v_B \in (1 - \pi, \pi)$, the Incumbent ticket wins *one* seat, and
2. if $v_A + v_B \in [\pi, 1]$, the Incumbent ticket wins *two* seats,

where the parameter $\pi \in \left[\frac{1}{2}, 1\right]$ is the electoral *quota*, and reflects the proportionality of the system. In real-world contexts, $\pi = \frac{2}{3}$ is called the *D'Hondt* method, and it is used (among others) in Brazil and Denmark. The variant with $\pi = \frac{3}{4}$ is called the *Sainte-Laguë* method, and it is used among others in Iraq, Sweden and Germany. A higher value of π increases the amount of excess support that one party must accumulate over the other in order to claim the second seat.

Allocation of Seats Within Parties. If $v_A + v_B \in (1 - \pi, \pi)$, so that the Incumbent ticket wins a single seat, only one representative is reelected. Which representative depends on *both* the party leadership's list assignment, *and* the distribution of preference votes within the Incumbent ticket. Specifically, if the party wins only a single seat, and the Incumbent ticket $\{ij\}$ assigns priority to representative i, then i is reelected if and only if:

$$\frac{v_i}{v_i + v_j} \geq \eta,$$

where $\eta \in \left[0, \frac{1}{2}\right]$ reflects the *flexibility* of the list: thus, if representative j is second-ranked, she is reelected if and only if her share of the party's total preference votes exceeds $1 - \eta$. The case $\eta = 0$

[7] The assumption of constant marginal effect of a single 'yes' vote on the bill's probability of passage saves notation, but plays no role on our results.

corresponds to a closed list, i.e., completely inflexible, in which case the first-ranked representative is always reelected in the event that the party wins a single seat. The case $\eta = \frac{1}{2}$ corresponds to an open list, i.e., completely flexible, in which preference votes wholly determine which of the two representatives is reelected when the party wins a single seat.

Payoffs. A constituency voter obtains a payoff θ if the policy agenda is implemented, and zero otherwise. Hence, her expected per-period payoff is $q\theta$, where we recall that q is the equilibrium probability with which the policy passes.

Both incumbents and the leadership care about winning district-level elections, in addition to policy outcomes in both periods. We assume that the party leadership derives a value G from the successful passage of the bill, and normalize to one its payoff from keeping each seat. Its per period expected payoff is then:

$$u_L(q, e) = qG + e(A) + e(B),$$

$G > 0$ could reflect a context in which the party leadership has agenda-setting authority and has sponsored the national policy. The policy could benefit the leadership ($G > 0$) but nonetheless generate a negative net payoff ($\theta < 0$) in a given district; achieving success on a *national* legislative strategy often requires party leaders to partly overlook its local consequences. This is especially likely if the district is not considered electorally marginal.[8] Alternatively, $G < 0$ could reflect a context in which the party is outside the national governing coalition and wishes to obstruct the government's agenda. While we subsequently clarify how our model can accommodate either view, for presentational ease we assume that $G > 0$, so that the leadership prefers the policy to succeed.

Each politician values her own re-election, which yields a private office rent R. Politicians also care about policy outcomes. Each incumbent and opposition legislator may be *aligned* ($\tau = 0$) with her voters, or instead *mis-aligned* ($\tau = 1$) with her voters. Each incumbent's type is independently drawn, and known only to herself. We denote by μ the common prior that an incumbent is aligned, by $\hat{\mu}_i(\text{t})$ the posterior conditional on vote tally t, and by $\mu_O = 1/2$ the common prior that an opposition candidate is aligned. An aligned politician shares her constituency's policy preferences, while a mis-aligned politician derives an additional payoff $\tau \in \{-b, b\}$ from the passage of the policy that is unrelated to its local consequences. For a mis-aligned incumbent, $\tau = b > 0$, reflecting that the incumbent intrinsically values passing the leadership's agenda. By contrast, a mis-aligned opposition politician derives the value $\tau = -b$ if the policy passes, reflecting a prim-

[8] We are grateful to Benoit Crutzen, who encouraged us to clarify this point.

itive hostility to the governing party's legislative agenda. A politician's expected payoff at each date is then given by:

$$u_i(q, e, \theta; \tau) = q(\theta + \tau) + Re(i)$$

The structure of equilibrium strategies, but not our substantive results, can be indexed according to whether the prior reputation of the Incumbent team μ is more favorable than the Opposition team ($\mu \geq 1/2$) or instead less favorable ($\mu < 1/2$). To streamline the exposition, our benchmark presentation focuses on the former context: we later fully extend all of our results to the case of $\mu < 1/2$.

Assumption 1. $\mu \geq 1/2$.

In addition to her direct payoffs from the implementation of the party's primary legislative agenda, each voter derives a per-capita incremental payoff $S > 0$ from having an aligned representative rather than a mis-aligned representative. Thus, a voter's per period payoff when representatives i and j are in office is given by:

$$u(q, \theta, \tau_i, \tau_j) = \begin{cases} 2S + q\theta & \text{if both representatives are aligned} \\ S + q\theta & \text{if one representative is aligned} \\ q\theta & \text{if neither representative is aligned.} \end{cases} \quad (1)$$

S captures the myriad ways in which aligned representative can champion constituency interests beyond voting on the policy agenda. Examples include inserting favorable amendments into secondary bills, using gate-keeping powers in committee to promote certain bills and forestall progress on others, and delivering government contracts and grants to local firms. It may also reflect individual bill initiation and sponsorship, which Däubler, Bräuninger and Brunner (2016) show is positively related to an incumbent's personal vote in Belgium.

Voting behavior. Recall that a fraction α of voters are *committed* to supporting only Incumbent candidates, while a fraction α are committed to supporting only Opposition candidates. We assume that these voters randomize uniformly over the two candidates from the party to which they are committed.[9] *Uncommitted* voters, by contrast, use first-period behavior to guide their choice both *between* and *within* party lists.

[9] The role of committed voters is to ensure that the division of preference votes within a party's list is a random variable. Lemma 2 shows that any uncommitted voters that support an Incumbent candidate always vote for the candidate with the highest reputation for alignment. Absent committed voters, this candidate would therefore win all preference votes within the party's list, rendering the degree of list flexibility $\eta \in (0, 1/2)$ irrelevant for election outcomes. The model can accommodate alternative strategies rules for committed voters, e.g., in which they vote for the highest-ranked candidate on their preferred party's list, with probability one.

Technically, the assumption of a continuum of voters ensures that no uncommitted voter can ever be decisive for electoral outcomes. In single-member contexts, this observation motivates "sincere" voting, in which each voter casts her vote for whichever candidate she most prefers. We set out a tractable way to extend the the notion of sincere and instrumentally rational voting to multi-member contexts.

We assume that uncommitted voters formulate conjectures about incumbents' types and about about other voters' behavior (which in equilibrium will be correct) and compute the expected values V_A from casting a preference vote in favor of incumbent A, V_B from casting a preference vote in favor of incumbent B, and V_O from voting for an Opposition candidate. An uncommitted voter J therefore prefers to cast a preference vote for representative A if and only if:

$$V_A \geq \max\{V_O + \xi + \sigma_J, V_B\}, \tag{2}$$

where $\xi \sim U[-\frac{1}{2\psi}, \frac{1}{2\psi}]$ is an aggregate preference shock in favor of the Opposition party, and $\sigma_J \sim U[-\frac{1}{2\phi}, \frac{1}{2\phi}]$ is an individual-level preference shock in favor of the Opposition party that is independently drawn for each voter. We interpret σ_J as an individual's partisanship. Conversely, ξ reflects factors that may influence all voters, such as a last-minute revelation of scandal or impropriety. Similarly, an uncommitted voter J prefers to cast a preference vote for representative B if and only if:

$$V_B \geq \max\{V_O + \xi + \sigma_J, V_A\}, \tag{3}$$

and casts a preference vote for an Opposition candidate if neither (2) nor (3) holds.[10]

Timing. The interaction proceeds as follows.

1. The date-1 constituency value θ, and the alignment of each incumbent $\tau_A \in \{0, b\}$ and $\tau_B \in \{0, b\}$ are independently realized. Each representative $i \in \{A, B\}$ observes θ, and her own alignment τ_i.

2. *Date 1 Legislative Interaction*: each representative votes *aye* (y) or *nay* (n). This tally is observed by voters and the party leadership.

3. *List Assignment*: the leadership chooses a ballot order $\{AB\}$ or $\{BA\}$. Voters observe the list, and formulate values V_A, V_B and V_O.

4. *Election*: the random variables ξ and σ_J are realized, voters cast their ballots, and seats are allocated.

[10] If an uncommitted voter is indifferent between casting her preference vote for a set of candidates, we assume that she randomizes uniformly over the subset of these candidates with the highest posterior reputation of alignment.

5. *Date 2 Legislative Interaction*: the date-2 constituency value θ—as well as the alignment of any newly elected Opposition representatives—are independently realized. Each representative votes *aye* (y) or *nay* (n).

6. Payoffs are collected, and the interaction ends.

Equilibrium. We study symmetric sequential equilibria. Note that the symmetry is with respect to the labeling of the incumbent politicians, *not* their alignment. This ensures that all heterogeneity in behavior across co-partisans is solely a consequence of the different incentives of aligned and mis-aligned politicians in different electoral contexts. When multiple equilibria arise, we select the one that maximizes the differences in behavior across types—we refer to this type of equilibria as "separating."

Discussion. Our framework accounts for both legislative interaction and electoral politics as distinct sources of legislative incentives, as in Snyder and Ting (2003, 2005), Eguia and Shepsle (2015) and Polborn and Krasa (2017). However, voters' induced preferences over candidates take into account that policy outcomes result from the interaction between many representatives—most of them from other constituencies—as in Austen-Smith (1984), Austen-Smith (1986), and Polborn and Snyder (2017). In contrast with these contributions, we compare different types of multi-member districts, as opposed to solely focusing on single-district contexts. Morelli (2004) and Kselman (2017) also study legislative behavior in multi-member systems, but focus respectively on party formation and public finance outcomes.

Other interpretations of the policy context. For ease of interpretation, we assume that the party leadership values the passing of the policy at each date. However, our model can easily capture a context in which the incumbent legislators and their party leadership could prefer the policy initiative to *fail*. In that context, a vote *aye* is whichever vote—in favor or against the policy—that the leadership values, while a vote *nay* is similarly a vote against the leadership.

Other sources of discipline. In our framework, party leaders discipline incumbent politicians via their list assignment. In real-world contexts, party leaders have other tools for enforcing discipline, such as endorsement of primary challengers or recruitment strategies aimed at selecting a more ideologically homogeneous candidate pool (Carroll and Nalepa, 2016). Later, we discuss one such tool: removing a representative from the electoral ticket, rather than simply demoting her to a lower rank. We highlight how party leaders may nonetheless prefer to use demotions in the party list rather than outright removal from the ballot.

Other sources of dyadic representation. We focus on roll-call voting on national bills that have local consequences as a means for politicians to serve their constituents, and to cultivate a personal vote, i.e., "policy representation". In particular, a representative who represents the policy interests of her district supports the bill if and only if $\theta > 0$. In practice, politicians have a wider behavioral repertoire to build a reputation for constituency alignment: Samuels (1999) shows how Brazilian politicians design pork-barrel projects in a multi-member context to cultivate personal reputations, and similar patterns are uncovered by Golden and Picci (2008) in post-war Italy and Crisp and Desposato (2004) in Colombia. Bräuninger, Brunner and Däubler (2012) and Däubler, Bräuninger and Brunner (2016) show that bill initiation—not just voting—plays an important role for reputation building. We view these important channels as complementary to our own, and their value could be reflected in the term S—the incremental payoff from having an aligned representative independently of to their roll-call voting.[11]

3. Analysis

Second Period. We begin by deriving the behavior of elected representatives at the second (i.e., terminal) date. An aligned representative values the party agenda only inasmuch as it generates a positive surplus for her constituents, i.e., if $\theta > 0$. By contrast, a mis-aligned representative derives a value $\tau \in \{-b, b\}$ from the party agenda: whenever the agenda is passed, a mis-aligned Incumbent derives a value $\theta + b$ and a mis-aligned Opposition legislator derives a value $\theta - b$.[12] We therefore obtain:

Lemma 1. *In the second period:*
(i) an aligned *representative votes* aye *on the party agenda if and only if $\theta > 0$;*
(ii) a mis-aligned *representative from the Incumbent party votes in favor if and only if $\theta > -b$;*
(iii) a mis-aligned *representative from the Opposition party votes in favor if and only if $\theta > b$.*

Lemma 1 implies that constituency voters and the party leadership have directly conflicting preferences over the representatives' types: constituency voters prefer an aligned representative, while the opposite holds for the Incumbent party leadership. The reason is that the policy agenda in the second period is more likely to succeed when Incumbent legislators are mis-aligned: whenever $\theta \in [-b, 0]$ only a mis-aligned Incumbent representative would support it.

We next turn to the equilibrium choices of voters, party leaders, and incumbent representatives in the first date, with a focus on the key trade-off that shapes their decisions to support their party's agenda.

[11] More generally, however, they could be substitutes: representatives might focus on providing visible outputs like pork instead of policy representation. We are grateful to Thomas Däubler for this observation.

[12] The symmetry of the mis-aligned representatives' biases across parties does not play an important qualitative role in our analysis.

Electoral Outcomes and List Assignments. After observing first-period policy outcomes but prior to the election, the party leadership L constructs the party *list* assignment: either $\{AB\}$ or $\{BA\}$. To understand this choice, we next study how voters cast preference votes, and then, in light of this, how list assignment shapes representatives' behavior.

Probability of reelection. Using the voting calculus developed in expressions (2) and (3), we obtain the total preference votes accruing to the Incumbent ticket as a function of the aggregate popularity shock:

$$v_A + v_B = \alpha + (1 - 2\alpha)\left(\frac{1}{2} + \phi[\max\{V_A, V_B\} - V_O - \xi]\right) \tag{4}$$

Since ξ is an aggregate shock in favor of the Opposition party, smaller realizations of ξ imply a higher vote share for the Incumbent ticket. We can obtain threshold values ξ_1 and ξ_2—with $\xi_1 > \xi_2$—such that the Incumbent ticket wins (a) both seats whenever the aggregate shock is sufficiently small (i.e., $v_A + v_B \geq \pi$ if and only if $\xi \leq \xi_2$), (b) neither seat whenever this aggregate shock is sufficiently large (i.e., $v_A + v_B < 1 - \pi$ if and only if $\xi > \xi_1$), and (c) only one seat when the aggregate shock does not disproportionately favor either ticket ($\xi_2 < \xi \leq \xi_1$). Using Equation 4, we derive these thresholds:

$$\begin{aligned}\Pr(v_A + v_B \geq \pi) &= \Pr\left(\xi \leq \max\{V_A, V_B\} - V_O - \frac{\pi - 1/2}{(1 - 2\alpha)\phi} \equiv \xi_2\right)\\ &= \Pr(\textbf{both incumbents reelected}),\end{aligned} \tag{5}$$

and

$$\begin{aligned}\Pr(v_A + v_B < 1 - \pi) &= \Pr\left(\xi > \max\{V_A, V_B\} - V_O + \frac{\pi - 1/2}{(1 - 2\alpha)\phi} \equiv \xi_1\right)\\ &= \Pr(\textbf{neither incumbent reelected}).\end{aligned} \tag{6}$$

Notice that when the Incumbent ticket wins zero seats or both seats, neither the within-ticket share of preference votes nor the list assignment matter. When, instead, the Incumbent ticket wins only a single seat, the list assignment and the distribution of preference votes within the ticket jointly determine to whom this seat is awarded.

An uncommitted voter may support either an Opposition candidate, or an incumbent candidate. Suppose that the list assignment is $\{ij\}$, and moreover that uncommitted voters who choose to support one of the incumbent candidates prefer to cast a preference vote for first-ranked incumbent i, rather than second-ranked incumbent j. Since she is *both* first-ranked by the party leadership *and* awarded a majority of the preference votes within the ticket, incumbent i is reelected whenever the ticket secures at least one seat ($\xi \leq \xi_1$), while incumbent j is reelected only if the ticket secures both seats ($\xi \leq \xi_2$). Letting $p_i(\{ij\}|i)$ denote the total probability that incumbent i wins under list $\{ij\}$, given that uncommitted voters that support the Incumbent ticket cast preference votes for her, i's prospect of reelection is

$$p_i(\{ij\}|i) = \Pr(v_A + v_B > 1 - \pi) = \Pr(\xi < \xi_1),$$

and the second-ranked j's prospect of reelection is

$$p_j(\{ij\}|i) = \Pr(v_A + v_B > \pi) = \Pr(\xi < \xi_2).$$

Suppose, instead, that uncommitted voters who cast a preference vote for an incumbent candidate direct these votes to second-ranked incumbent j, rather than first-ranked incumbent i. This implies that every uncommitted voter who chooses the Incumbent ticket not only increases the ticket's total performance, i.e., $v_A + v_B$, but also raises the *relative* share of preference votes that go to candidate j, i.e., $\frac{v_j}{v_A+v_B}$. Recall that if $\frac{v_i}{v_A+v_B} < \eta$, the second-ranked candidate is elected despite candidate i being first-ranked on the list. Thus, candidate i is reelected only if this condition *fails*—using Equation 4, we obtain a third threshold ξ_η:

$$\left.\frac{v_i}{v_A + v_B} \geq \eta \right| \text{ uncommitted voters prefer } j \text{ to } i \iff \xi \geq V_j - V_O + \frac{1 - \frac{\alpha}{\eta}}{2\phi(1 - 2\alpha)} \equiv \xi_\eta. \tag{7}$$

In words: if the Incumbent ticket wins a single seat, a relatively unpopular (amongst uncommitted voters) first-ranked incumbent i is awarded the seat only if she does not receive too few preference votes compared to the more popular j. Equation 7 reveals that this can only happen if the Incumbent ticket as a whole *does not perform too well* among the uncommitted voters, which requires the Opposition preference shock ξ to be *large enough*.

The total prospect that the first-ranked incumbent i is awarded a seat when uncommitted voters prefer incumbent j over i is then

$$p_i(\{ij\}|j) = \Pr(\xi \leq \xi_2) + \Pr(\max\{\xi_\eta, \xi_2\} \leq \xi \leq \xi_1), \tag{8}$$

Paradoxically, a relatively unpopular first-ranked incumbent candidate therefore prefers the Incumbent ticket as a whole either to do *very* well—so that it wins both seats—or *barely* well enough to clinch a single seat without the second-ranked candidate displacing her.

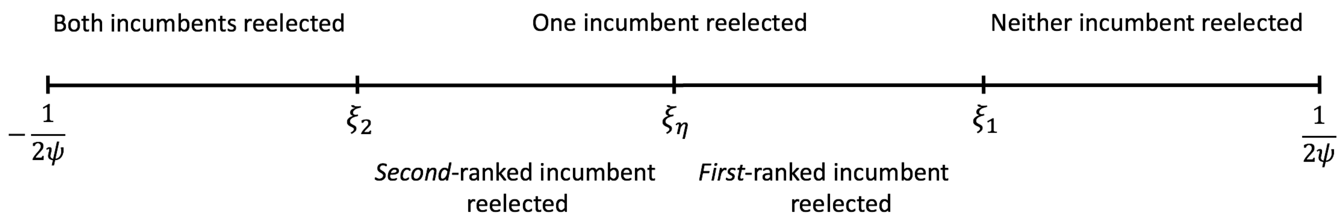

Figure 1 – Incumbent ticket's electoral support and the prospects of reelection of a relative unpopular first-ranked and relatively popular second-ranked incumbent.

The corresponding prospect that the second-ranked incumbent candidate is awarded a seat is

$$p_j(\{ij\}|j) = \Pr(\xi \leq \xi_2) + \Pr(\xi_2 < \xi \leq \min\{\xi_\eta, \xi_1\}). \tag{9}$$

Figure 1 illustrates the thresholds on Opposition support derived above, highlighting the non-monotonicity in the unpopular first-ranked incumbent's reelection prospects.

When η lies in an intermediate range $(\frac{\alpha}{2\pi}, \frac{\alpha}{2(1-\pi)})$, the threshold ξ_η lies between ξ_2 and ξ_1, as depicted in Figure 1. Intuitively, if the list is sufficiently inflexible (η very small), the second-ranked incumbent can never displace the first-ranked incumbent. In that case, our environment is strategically equivalent to a closed list. If instead the list is sufficiently flexible (η very large), whichever incumbent candidate receives the most preference votes is reelected, regardless of her list ranking, whenever the party wins only one seat. In that case, our environment is strategically equivalent to an open list. Since we explicitly consider these polar cases in a subsequent part of the paper, it is thus without loss of generality that we restrict attention to this intermediate degree of flexibility:

Assumption 2. $\frac{\alpha}{2\pi} < \eta < \frac{\alpha}{2(1-\pi)}$.

Notice that while we assume that committed voters randomize uniformly over candidates from the party to which they are committed, we could alternatively assume that they instead vote for whichever candidate is top-ranked, i.e., they follow the party leadership's cue. All of our subsequent results carry over to this setting: we need only modify the range of list flexibility η in Assumption 2, appropriately.[13]

Under Assumption 2, the probabilities that the incumbents are re-elected, given the list assignment $\{ij\}$ and that an uncommitted voter who casts a preference vote for an incumbent does

[13] We are grateful to Odilon Câmara for encouraging us to clarify this point. After summarizing voting behavior, in Lemma 2, we clarify the role that committed voters play in our framework.

so for the *second*-ranked j are then:

$$p_i(\{ij\}|j) = \frac{1}{2} + \psi(V_j - V_O) - \frac{\psi}{2\phi(1-2\alpha)}\left(1 - \frac{\alpha}{\eta}\right), \tag{10}$$

$$p_j(\{ij\}|j) = \frac{1}{2} + \psi(V_j - V_O) + \frac{\psi}{2\phi(1-2\alpha)}\left(1 - \frac{\alpha}{\eta}\right). \tag{11}$$

Notice that greater flexibility necessarily hurts a relatively unpopular first-ranked incumbent, since it raises the risk that she is displaced by her lower-ranked but more popular co-partisan.[14] In fact:

Observation 1. *A more popular second-ranked incumbent is more likely to be reelected than an less popular first-ranked incumbent if and only if the list is sufficiently flexible ($\eta > \alpha$).*

Assumption 1 ensures that a sufficiently popular second-ranked incumbent can nudge out a first-ranked incumbent to win a single seat. Expressions (10) and (11) reveal that this is more than an abstract possibility: it may be the most likely scenario. Whenever the party list is sufficiently flexible, a first-ranked unpopular incumbent faces a *lower* prospect of winning reelection than her second-ranked but more popular co-partisan.

Voting Decisions. Given an incumbent list order $\{ij\}$, uncommitted voters decide whether to cast a preference vote for one of the Incumbent candidates A or B, or instead for one of the Opposition candidates. Recall that we have a continuum of voters, so that no voter can ever be decisive for the outcome of the election. In this section, we nonetheless develop a calculus of voting that reveals a simple choice for uncommitted voters: if the preference shock ξ in favor of the Opposition party is large enough, an uncommitted voter prefers to vote for an Opposition candidate; otherwise, she prefers to vote for the Incumbent candidate with the highest posterior reputation of alignment.[15]

Since no voter can be decisive for the outcome, *any* voting behavior is consistent with optimality. If there were only a single representative (i.e., a single-member system) in a two-candidate contest, the election outcome turns on a single event: one candidate winning a majority of the votes. Thus, in the hypothetical event that a voter *could* have been decisive for the outcome, she would always prefer to cast her ballot in favor of the candidate she most prefers. This motivates a natural restriction to *sincere* voting by uncommitted voters in single-member contexts.

[14] Each individual incumbent's prospect of reelection is independent of the electoral system proportionality, π. Of course, this is conditional on Assumption 1, which implicitly links list flexibility η and electoral system proportionality π.

[15] This corresponds to a notion of "sincere" voting in which a voter simply votes for her favorite candidate (inclusive of party preferences reflected in the stochastic shocks σ and ξ). As we detail below, however, it is difficult to define sincere voting in a multi-member context, which is why we develop an explicit voting calculus.

In a multi-member context, however, the consequences of a preference vote for either Incumbent representative depend on how other voters in the same district cast their ballot. To see this, notice that a voter's choice could be decisive for the *inter-party* contest, by changing the number of seats awarded to each party, or for the *intra-party* contest, by determining whether the first- or second-ranked Incumbent wins the single seat awarded to this ticket. Thus, the same preference vote could result in different election outcomes, depending on how other voters cast their ballots. For example, a preference vote for the second-ranked Incumbent candidate may end up advancing the election prospects of only the first-ranked candidate.

In order to account for these features of multi-member elections, we suppose that an uncommitted voter formulates values from supporting either of the Incumbent ($\max\{V_A, V_B\}$), or Opposition (V_O) by asking: *if* I were decisive, what is the relative prospect that I would be decisive for each of up to three possible events:

1. the Incumbent ticket wins the second seat ($\xi = \xi_2$),
2. the Incumbent ticket wins the first seat ($\xi = \xi_1$), or
3. the first- or second-ranked incumbent is reelected, conditional on the Incumbent ticket winning only one seat ($\xi = \xi_\eta$).

With uniform preference shocks, the likelihood of each critical realization is—conditional on being positive—the same.[16] In the Appendix, we show that, as a consequence, the continuation payoffs weigh these pivotal events equally. Recall that $\hat{\mu}_i$ is incumbent i's perceived probability of alignment, at the end of the date-1 interaction. Our next Lemma highlights useful properties of voter behavior.

Lemma 2. *$V_i \geq V_j$ if and only if $\hat{\mu}_i \geq \hat{\mu}_j$. Moreover,* $\lim_{\kappa\to\infty} \max\{V_i, V_j\} - V_O \propto \lim_{\kappa\to\infty}(\hat{\mu}_i + \hat{\mu}_j) - 2\mu_O$.

Lemma 2 states that our voting calculus generates a simple and intuitive decision rule: modulo preference shocks, uncommitted voters prefer to vote for the incumbent candidate with the highest reputation of alignment. It further highlights that an uncommitted voter's difference in payoff from supporting her most-preferred incumbent candidate or an Opposition candidate is—to a first approximation—proportional to the difference between the tickets' *average* reputation.[17]

Leadership choice. We can now study the leadership's list assignment as a function of the vote tally, $\mathsf{t} \in \{\mathsf{y}, \mathsf{n}\}^2$. In a symmetric equilibrium, if both representatives vote the same way—either both

[16] Observe that when the perceived alignment of the first-ranked candidate exceeds the second candidate's, no voter can be pivotal for the intra-party contest.

[17] The proof of Lemma 2 establishes that $\lim_{\kappa\to\infty}(\hat{\mu}_i + \hat{\mu}_j)$ is well-defined.

vote *aye* or both vote *nay*—party leaders and voters assign each representative the same posterior assessment of alignment. Since the leadership is indifferent between the candidates, it resolves indifference by mixing uniformly over list assignments $\{AB\}$ and $\{BA\}$.[18]

Suppose, instead, that one representative i opposes the leadership, while the other supports it. In the Appendix, we show that in a symmetric pure strategy equilibrium, a split record always indicates that the representative who supported the leadership is mis-aligned, and the representative that opposed the leadership is aligned.[19] A representative who is relatively mis-aligned with constituents is less likely to prioritize his voters' needs over party goals, and is therefore more valuable to the party leadership in the second period. And—given the uniform aggregate preference shock—placing this representative at the top of the list does not weaken the expected share of preference votes accruing to the Incumbent team.[20] We therefore obtain the leadership's list strategy.

Lemma 3. *Whenever agents believe that representative i is* aligned*, and that the other representative j is* mis-aligned*, the leadership ranks first the representative with the lower posterior alignment, i.e., chooses the list $\{ji\}$.*

The reason is that the leadership values the reelection of the relatively unpopular incumbent, whose reputation for alignment with voters is lower.

Our results imply that incumbents who vote more frequently with their party receive fewer preference votes (see Crisp et al., 2013).[21] Alternatively stated, Lemma 3 implies that developing a personal reputation for alignment with one's district comes at the cost of a worse list assignment. This cost is at the heart of the trade-off that incumbent legislators face, to which we now turn.

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[18] Uniform randomization over list assignments is a necessity of a symmetric equilibrium, despite the leadership's indifference.

[19] The separating equilibrium we characterize is in pure strategies. In the Appendix, we characterize a symmetric equilibrium for $\mu < 1/2$ in which, in an intermediate interval of θ, the incumbents mix.

[20] In Section 8 we discuss some ways in which the position of the representative on the ballot could affect the total share of preference votes to the ticket, and how this affects both leadership and representatives' incentives.

[21] They are also consistent with Depauw and Martin (2009) and Crisp et al. (2013)'s finding that voting in line with the party and obtaining more preference votes in the previous election are correlated with better list assignments in future electoral contests. The reason is that our model describes list assignments of two incumbents with the *same initial reputation*, but distinct voting records, after an initial term in office. It does not exclude the possibility that, after observing the relative share of preference votes won by each candidate in the election, party leaders respond by moving more popular incumbents to higher locations in the ballot in subsequent elections. We consider incentives to promote more popular incumbents in an extension that we outline in Section 8.

First Period Policy Outcomes

We show that if κ is not too small there is a unique separating symmetric equilibrium. Recall our Assumption 1 that $\mu > 1/2$.[22] We show that the unique separating symmetric equilibrium takes a simple threshold form: each type votes *aye* if and only if θ exceeds a critical value $\theta^*_\tau(\kappa, \eta)$, with $\theta^*_b(\kappa, \eta) = \theta^*_0(\kappa, \eta) - b$. As κ becomes large, the equilibrium thresholds (θ^*_b, θ^*_0) converge to quantities that can be characterized analytically.

Proposition 1. *For κ not too small, there exists a unique separating symmetric equilibrium. In this equilibrium, an aligned representative votes* aye *if and only if $\theta \geq \theta^*(\kappa, \eta)$, and a misaligned representative votes* aye *if and only if $\theta \geq \theta^*(\kappa, \eta) - b$, where*

$$\lim_{\kappa \to \infty} \theta^*(\kappa, \eta) = \frac{RS\psi}{\chi} \left[(2\mu - 1)(\mu - 1/2) + \frac{1}{2\phi S(1 - 2\alpha)} \left(1 - \frac{\alpha}{\eta} \right) \right]$$

When the representatives both vote aye *or both vote* nay*, the leadership randomizes over the list assignments; if only one representative votes* aye*, the leadership assigns the list priority to that representative.*

To construct the equilibrium thresholds, consider the problem facing a representative $i \in \{A, B\}$ of type τ, who learns the constituency value of the policy θ, and cannot perfectly anticipate the behavior of her co-partisan.

The representative anticipates that if both representatives support their party's agenda (vote *aye*)—they will subsequently share the same posterior perceived alignment, i.e., $\hat{\mu}(\mathsf{y}, \mathsf{y})$, so that (for κ large enough) uncommitted voters' relative value of voting for either incumbent $i \in \{A, B\}$ is

$$V_i(\mathsf{y}, \mathsf{y}) - V_O \propto \frac{1}{2} S(\hat{\mu}(\mathsf{y}, \mathsf{y}) - \mu_O) + \frac{1}{2} S(\hat{\mu}(\mathsf{y}, \mathsf{y}) - \mu_O) = S(\hat{\mu}(\mathsf{y}, \mathsf{y}) - 1/2), \tag{12}$$

where we recall that $\mu_O = 1/2$. The reason is that a voter assesses an equal relative prospect that her preference vote in favor of i will make the difference between (a) the first-ranked incumbent (with perceived alignment $\hat{\mu}(\mathsf{y}, \mathsf{y})$) and the second-ranked opposition candidate (with perceived alignment μ_O) or (b) the second-ranked incumbent (also with perceived alignment $\hat{\mu}(\mathsf{y}, \mathsf{y})$) and the first-ranked opposition candidate (also with perceived alignment μ_O).

Likewise, if both representatives oppose their party's agenda (vote *nay*), they will subsequently share the same posterior perceived alignment, i.e., $\hat{\mu}(\mathsf{n}, \mathsf{n})$, so that (for κ large enough)

[22] We characterize the unique symmetric equilibrium for $\mu \leq \frac{1}{2}$ in the Appendix.

uncommitted voters' relative value of voting for either incumbent $i \in \{A, B\}$ is

$$V_i(\mathsf{y},\mathsf{y}) - V_O \propto \frac{1}{2}S(\hat{\mu}(\mathsf{n},\mathsf{n}) - \mu_O) + \frac{1}{2}S(\hat{\mu}(\mathsf{n},\mathsf{n}) - \mu_O) = S(\hat{\mu}(\mathsf{n},\mathsf{n}) - 1/2).$$

In the Appendix, we show that as κ tends to infinity, a unified vote tally becomes virtually uninformative, so $\hat{\mu}(\mathsf{y},\mathsf{y})$ and $\hat{\mu}(\mathsf{n},\mathsf{n})$ approach μ.

If instead the representative i votes against the policy, but her co-partisan j supports the policy, the party leadership and voters conclude that i is aligned with probability one and j is aligned with probability zero. The split record (n,y) ensures that the leadership assigns her the lowest rank in the list, i.e., chooses the ballot order $\{ji\}$, but also that uncommitted voters, conditional on voting for the Incumbent ticket, cast their preference votes for her. Uncommitted voters' value of doing so, is

$$V_i(\mathsf{n},\mathsf{y}) - V_O \propto \frac{1}{3}\underbrace{S(0-1/2)}_{\xi=\xi_1} + \frac{1}{3}\underbrace{S(1-0)}_{\xi=\xi_\eta} + \frac{1}{3}S\underbrace{(0-1/2)}_{\xi=\xi_2} = 0. \tag{13}$$

To understand this expression, recall that in the case where the relatively popular incumbent candidate is second-ranked, an uncommitted voter may be decisive in three ways. *First*, she could be decisive for awarding the Incumbent ticket the first seat ($\xi = \xi_1$), in which case the top-ranked incumbent—who is believed to be aligned with probability zero—is elected instead of an Opposition candidate, who is aligned with probability $1/2$. *Second*, she could be decisive for the single Incumbent seat to be awarded to the second-ranked incumbent i—who is believed to be aligned with probability one—at the expense of the first-ranked incumbent ($\xi = \xi_\eta$). *Third*, she could be decisive for awarding the second seat to the Incumbent ticket, in which case she changes the expected alignment of her date-2 politicians from $1+1/2$ to $1+0$, since both incumbents are retained ($\xi = \xi_2$).

The final consideration for the incumbent i is how her vote affects her list position: if she votes against a policy that she expects his co-partisan to support, she is demoted to the second rank on the list, which changes her reelection probability by

$$-\frac{1}{2\phi(1-2\alpha)}\left(1 - \frac{\alpha}{\eta}\right), \tag{14}$$

which could be negative ($\alpha < \eta$) or positive ($\alpha \geq \eta$). Thus, an approximation of the relative change in representative i's probability of reelection from supporting the policy—when she expects her co-partisan j to do the same—can be obtained as the difference of (12) and the sum of (13) and (14):

$$\underbrace{S\,(\mu - 1/2)}_{\text{inter-party}} + \underbrace{\frac{1}{2\phi(1-2\alpha)}\left(1 - \frac{\alpha}{\eta}\right)}_{\text{intra-party}}. \tag{15}$$

Reputational considerations enter this expression through both the inter-party and the intra-party contests.

Inter-party contest. Suppose that a representative anticipates that her co-partisan will support the party's agenda. If she, too, votes in favor, voters assess that each incumbent is aligned with probability $\mu(\mathsf{y},\mathsf{y}) \xrightarrow[\kappa\to\infty]{} \mu$. If, instead, she opposes the agenda when she anticipates that her co-partisan will support, voters believe that she is aligned with probability one, and that her colleague is aligned with probability zero. Yet, in spite of improving her *personal reputation*, the net consequence for the *collective reputation* of the party is negative: the average evaluation of the ticket reverts from μ to $\frac{1}{2}$. Since both politicians appear on the same ballot, and voters evaluate incumbents on the list collectively—not merely individually, individual reputation-building at the expense of other team members may come at too great a cost.

Intra-party contest. Suppose that a representative anticipates that her co-partisan will support the party's agenda. If she votes against, there are two consequences. First, the party leadership punishes her by demoting her to the lower rank on the party's list. Second, voters infer that she is aligned and uncommitted voters will cast their preference votes in her favor. The first force encourages her to support the party line, while the second force encourages her to oppose it. The net consequence depends on the degree of list flexibility: if the list is sufficiently flexible ($\eta > \alpha$), intra-party considerations push a representative to oppose; if the list is instead relatively inflexible ($\eta < \alpha$), intra-party considerations further discipline incumbents against pandering to voters. The reason is that list flexibility and the relative value of being first-ranked push in opposite directions: as the value of the priority list assignment increases, so too does the value of cultivating the favor of party leaders.

4. Forces Shaping Legislative Cohesion in Open Systems.

We now explore how changes in primitives shape legislative behavior, by studying properties of threshold θ^*, evaluated at its analytical limit.[23] We define legislative cohesion as the ex-ante probability that an incumbent representative votes with their party leadership.

[23] The comparative statics hold for sufficiently large κ; evaluation at the analytical limit is necessary for tractability.

Changes in list flexibility, and the share of committed voters. Consistent with conventional wisdom, (e.g., Carey, 2007) our framework predicts that legislative cohesion will be higher in countries with less flexible lists.

Corollary 1. *Legislative cohesion increases when*

i list flexibility η decreases, or

ii the fraction of committed voters, i.e., α, increases.

Incumbents value the top slot on the party list more as the list becomes less flexible, since the marginal value of preference votes declines; in turn, they are more prone to toe the party line instead of cultivate a personal vote through obstruction. Similarly, a larger share of committed voters, i.e., who cast preference votes without taking into account their beliefs about alignment, dilutes the pool of uncommitted voters and diminishes their relative influence amongst the set of voters that support the Incumbent ticket.

Changes in polarization amongst uncommitted voters. Suppose that inter-party polarization amongst uncommitted voters increases, i.e., ϕ decreases. Should we expect representatives to toe the party line more, or less? The answer depends both on list flexibility η and the proportion of uncommitted voters.

Corollary 2. *If polarization amongst uncommitted voters increases, (i.e.,* lower *ϕ),*

i. legislative cohesion increases *if the list is sufficiently inflexible $\eta < \alpha$,*

ii. legislative cohesion decreases *if the list is sufficiently flexible $\eta > \alpha$.*

A more polarized electorate shifts the locus of electoral competition from *between* parties to *within* parties. The reason is that uncommitted voters increasingly choose their preferred party on the basis of factors that are unrelated to perceptions of alignment, and which are instead captured by the partisan preference shock σ. It also reduces the prospect that either party sweeps the board by winning a vote share in excess of π, and thus both legislative seats. From an incumbent's perspective, this heightens the relative importance of the intra-party contest as the key to her electoral survival.

Nonetheless, the increase in partisan polarization does not affect uncommitted voters' preferences over candidates within the same party list. Thus, the consequences for legislative behavior depend on list flexibility. If the list is not too flexible, more intra-party competition translates into more competition for the favor of the party leadership, encouraging representatives to toe the party line. If the list is sufficiently flexible, however, more intra-party competition translates into more competition for preference votes, encouraging representatives to cultivate preference votes by opposing the party line.

These results identify list flexibility as a crucial moderating variable in the relationship between polarization and legislative cohesion. As a consequence, empirical analyses relying on the traditional dichotomy between open and flexible versus closed lists suffer from a fundamental problem of mis-specification: because the set of open and flexible lists masks enormous variation, Corollary 2 implies that a researcher could incorrectly conclude a null effect.

5. Legislative Cohesion Across Electoral Institutions

Our framework allows us to compare different varieties of list PR systems—including the polar cases of open and closed lists. However, we can also adapt our model to accommodate single-member district (SMD) contexts, in which only one incumbent appears on the ballot.[24]

Under closed list systems, voters may cast a vote for a party list but they cannot cast a vote for a particular candidate within the list. In our framework, this is equivalent to a completely inflexible list, i.e., $\eta = 0$:[25] under closed lists (CLPR) the party's rank-order is never overturned and the first-ranked candidate gets reelected whenever the party wins at least one seat.

In a single-member context (SMD), the Incumbent ticket is a single candidate. While, the leadership may play an active role in candidate selection and recruitment, there is no prospect of moving a candidate up or down the list: the party either wins the single seat or it wins no seats.

Proposition 2. *In equilibrium:*

1. *legislative cohesion is* strictly higher *under flexible lists than under CLPR, and converges to CLPR as list flexibility η converges to $\frac{\alpha}{2\pi}$, and*
2. *there exists a threshold value of flexibility η^S such that legislative cohesion is* strictly lower *under flexible lists than under SMD if and only if $\eta > \eta^S$.*

Consider a single-member setting (SMD). When a politician in district A panders by obstructing his party's agenda, his actions *indirectly* affect how his co-partisan from another district (B) is assessed by his own voters—just as in the multi-member context. The difference, however, is that the representatives are accountable to distinct sets of voters: the representative in A does not

[24] To ensure a proper comparison, we divide the constituency into two identical districts and appropriately scale payoffs.

[25] In fact, in our framework all systems with $\eta \in [0, \frac{\alpha}{2\pi}]$ are observationally equivalent in terms of incentives and behavior.

directly care about his co-partisan's reputation—whether he is reelected or not depends solely on how he is evaluated, relative to the challenger in his own district. In other words, there is no intra-party contest.[26] Relative to flexible lists, this raises the relative value of pandering through obstruction in single-member contexts, since it eliminates a penalty that she would incur in the multi-member setting, where voters assess politicians as a team.

There is, however, a second channel that encourages obstruction under flexible lists: in the event that the ticket wins only one of the two seats, one incumbent's reelection necessarily comes at the other's cost. In this case—unlike in SMD—the representatives are implicitly competing directly with one another for the single seat.

In summary, relative to SMD, a flexible list system generates a penalty to collective reputation that diminishes incentives to cultivate a personal vote, but it also creates a form of direct competition amongst co-partisans that increases incentives to cultivate a personal vote. In sufficiently flexible list contexts, i.e., η sufficiently large, the latter effect dominates, *lowering* legislative cohesion.

6. Dyadic Representation under List PR

We now study the relationship between elected representatives and their local constituents under open lists. In particular, we ask: *to what extent do different degrees of list flexibility induce incumbents to act as agents for their constituency on legislative decisions*? Following Miller and Stokes (1963) and Ansolabehere and Jones (2011), we refer to this as *dyadic representation*.

The quality of dyadic representation is captured in our framework by voters' expected first-period policy payoffs. Notice that our equilibrium suggests two possible detriments to effective dyadic representation:

1. *obstructionism*—pandering to voters by opposing projects that would benefit the constituency, in the hope of improving one's personal reputation and therefore attract preference votes.
2. *rubber-stamping*—pandering to party leaders by supporting projects that would hurt the constituency, in the hopes that party leaders interpret this loyalty as a willingness to toe the party line in the future, to be rewarded with a favorable list assignment.

Proposition 3. *There exists a threshold η^* such that a more flexible list worsens dyadic representation if and only if $\eta \geq \eta^*$.*

[26] Note that while some SMD systems feature intra-party contests in the form of primary elections, these do not pit two incumbents from the same district in the pervious legislative session against one another, which is the crucial feature of an MMD system. In an SMD primary election, *at most* one of the competitors in a primary election is a sitting incumbent. Thus, the nature of the intra-party contest is distinct from the MMD context.

Proposition 3 directly contradicts the common scholarly wisdom that list flexibility improves the representation of local geographic interests (Ames, 1995; Carey and Shugart, 1995; Crisp et al., 2004; Hallerberg and Marier, 2004). More generally, it shows that the quality of dyadic representation is strictly quasi-concave in the degree of list flexibility, η.

A more flexible list *always* loosens the grip of party leaders on the reelection incentives of incumbent legislators. If this loss of control represented a transfer of value from parties to voters, the common wisdom would be correct. This intuition, however, is incomplete: While a more flexible list liberates representatives from the need to engage in rubber-stamping it also forces them to chase preference votes by pandering to their own electorates via obstruction. Thus, a reduced incentive to oppose policies that are harmful to their constituents comes at the expenses of an increased incentive to also oppose those that are *valuable* to them.

Starting from a closed list, the first-order effect may be to reduce rubber-stamping, in which case dyadic representation improves with a flexible list. But, eventually, more list flexibility aggravates pandering incentives to such an extent that voters and party leaders alike suffer as a consequence. Thus, an implication of Proposition 3 is that voters partially value delegating the control of electoral incentives to an agent—i.e., party leaders—whose preferences over representatives—i.e., aligned versus mis-aligned—are completely opposed to their own.

In the Appendix, we show that η^* increases as the fraction of committed voters, i.e. α, increases. The reason is that as α increases, there are less voters who reward candidates for cultivating a personal reputation for alignment. A larger share of committed voters in the electorate makes the vote share *across* parties less sensitive to individual politicians' behavior, simultaneously projecting the locus of competition inside parties *and* shifting that competition in favor of appealing to party leaders. Raising the list flexibility re-calibrates the intra-party contest in favor of uncommitted voters, which is more valuable when these voters are relatively more scarce.

An implication of this result is that moving from closed to open list is more likely to be valuable in divided societies, where parties are organized along religious or cultural cleavages and a large share of voters cast their ballots solely on the basis of these cleavages (i.e., large α).

Corollary 3. *Closed (i.e., completely inflexible) lists yield strictly better dyadic representation than open (i.e., completely flexible) lists if and only if one of the following holds:*

1. *voters place a sufficiently large value S on aligned representatives, or*
2. *politicians' office-holding value R is sufficiently large.*

The Corollary highlights a stark result: in spite of the additional control that preference votes under open list PR offers voters, under arguably mild conditions dyadic representation of local interests is intrinsically better under closed list PR . The reason is that the increased intra-party competition for preference votes encourages representatives to excessively obstruct policies—including those that would benefit constituents, in an attempt to build personal reputation that distinguishes them from their co-partisans.

7. Extension: District Magnitude and the Personal Vote

In our benchmark setting, voters in a geographic district are represented by two legislators. More generally, however, there is significant variation within and across countries in *district magnitude*—the number of representatives that serve the same geographic constituency. For example, while Chile had a district magnitude of two in all districts until 2015, district magnitude now ranges between three and eight representatives in elections to the lower house. In Brazil's lower house, district magnitude ranges from eight to seventy, while in Sweden district magnitude in the unicameral national parliament ranges from two to thirty-eight.

Larger districts are thought to exacerbate reputation-building incentives in open-list contexts. Most famously, Carey and Shugart (1995) argue that "as the number of copartisans from which a given candidate must distinguish herself grows, the importance of establishing a unique personal reputation, distinct from that of the party, also grows". (Carey and Shugart, 1995, 430)

To examine this conjecture, we extend our benchmark model to a context with an arbitrary district magnitude, in which there are initially $n \geq 2$ co-partisan incumbents. In a Supplemental Appendix, we characterize a symmetric equilibrium that generalizes our two-member benchmark to n representatives. We use this characterization to address the following question: does higher district magnitude foster greater legislative cohesion?

Proposition 4. *For each type $\tau \in \{0, b\}$, for κ not too small, there exists a symmetric equilibrium characterized by thresholds $(\underline{\theta}_\tau(\kappa; n), \overline{\theta}_\tau(\kappa; n))$ satisfying:*

$$\underline{\theta}_b(\kappa; n) < \overline{\theta}_b(\kappa; n) < \underline{\theta}_0(\kappa; n) < \overline{\theta}_0(\kappa; n),$$

such that a type-$\tau \in \{0, b\}$ representative:

i. votes nay *if $\theta \leq \underline{\theta}_\tau(\kappa; n)$,*

ii. randomizes over nay *and* yea *if $\theta \in (\underline{\theta}_\tau(\kappa; n), \overline{\theta}_\tau(\kappa; n))$, and*

iii. votes aye *if $\theta \geq \overline{\theta}_\tau(\kappa; n)$.*

In words: if the constituency value θ is very low, a representative votes *against* the policy; if the constituency value θ is very high, a representative votes *in favor* of the policy; for intermediate values, she plays a mixed strategy.

To gain insight into the consequences of greater district magnitude, we highlight how these thresholds (approximated analytically via $\kappa \to \infty$) vary under *completely flexible* (i.e. open) lists versus flexible lists (i.e., not completely flexible), as district magnitude increases. As in our benchmark setting, we define $\overline{\theta}_\tau(n) \equiv \lim_{\kappa\to\infty} \overline{\theta}_\tau(\kappa; n)$, with $\underline{\theta}_\tau(n)$ defined in the corresponding manner.

Proposition 5. *If the list is completely flexible (i.e., open):*

$$\lim_{n\to\infty} \overline{\theta}_0(n) = \frac{R\psi}{2\phi(1-2\alpha)}, \; \lim_{n\to\infty} \underline{\theta}_0(n) = 0, \;\; \lim_{n\to\infty} \overline{\theta}_b(n) = -b, \;\; \lim_{n\to\infty} \underline{\theta}_b(n) = -b. \tag{16}$$

If the list is neither completely flexible nor completely inflexible:

$$\begin{aligned}
\lim_{n\to\infty} \overline{\theta}_0(n) &= \frac{R\psi}{2\phi(1-2\alpha)}, & \lim_{n\to\infty} \underline{\theta}_0(n) &= -\frac{R\psi}{2\phi(1-2\alpha)}, \\
\lim_{n\to\infty} \overline{\theta}_b(n) &= -\frac{R\psi}{2\phi(1-2\alpha)} - b, & \lim_{n\to\infty} \underline{\theta}_b(n) &= -\frac{R\psi}{2\phi(1-2\alpha)} - b.
\end{aligned} \tag{17}$$

To interpret this result—including the comparison between different variants of flexibility, consider the problem faced by a representative, who learns that the constituency value of the project is θ, and anticipates a fraction p of her co-partisans will vote against the project. For fixed p, how does the value of dissent change with larger district magnitude?

Suppose, for example, that $p = 0$—which holds at the threshold $\overline{\theta}_0(n)$. In this case, a representative anticipates that all of her co-partisans will support their party's agenda. Her own vote in favor makes her indistinguishable from her colleagues, whereas a dissenting vote uniquely positions her to receive the preference votes of uncommitted voters, ensuring that so long as the party wins *at least* one seat, she will secure reelection.

As district magnitude increases, the representative's relative value from uniquely differentiating herself *increases*, raising her value from dissent. As n grows large, the equilibrium threshold remains strictly positive and bounded away from zero—the threshold that an aligned representative would use in the absence of electoral incentives. Thus, higher district magnitude lowers legislative cohesion through this channel. This is true for *both* flexible *and* open-list contexts.

Suppose, instead, that $p = 1$, which holds at the threshold $\underline{\theta}_b(n)$. In this case, a representative anticipates that all of her co-partisans will oppose the party's agenda in the hopes of building personal reputation (as well as for policy reasons). The consequences of higher district magnitude vary with the electoral rule.

Completely flexible (i.e., open) list. When the representative opposes the agenda, she may draw sufficient preference votes to secure reelection, regardless of her ranking in the list. Moreover, the party leadership has no way to punish her by sending her to the bottom of the list, since only preference votes determine the order of election. With the possibility of advancement in the list, and no prospect of sanction for her obstruction, the representative is encouraged to engage in more obstruction.

As district magnitude increases, however, the plethora of co-partisans that are trying to build personal reputation crowd one another, lowering the relative anticipated gain in preference votes. The net effect is to lower the role that electoral incentives play in the representative's decision. At large levels of district magnitude, when a representative anticipates that her co-partisans will obstruct, her own voting decision is increasingly driven by her policy motives.

Neither completely *flexible* (open), nor completely *inflexible* (closed). When the representative opposes the agenda, she may draw sufficient preference votes to secure reelection. Contrary to an open list, however, the party leadership has a powerful tool to sanction her: placing her at the bottom of the list. In the event that the representative fails to receive enough preference votes to secure election, she is in a comparatively worse position in a flexible list system than she would be in an open-list system.

As district magnitude increases, this trade-off resolves increasingly in favor of obstruction, but at a diminished rate vis-a-vis the open list context. The limiting thresholds do *not* converge to sincere voting—as they would in the open-list case. Rather, a wedge remains that is proportional to $-\frac{R}{1-2\alpha}$, reflecting that while the leadership's power diminishes with higher district magnitude, it retains a degree of control that is *not* fully dissipated by higher levels of district magnitude.

Our findings provide both qualified and nuanced support for the classic argument—most notably advanced by Carey and Shugart (1995)—that higher district magnitude in open-list PR lowers legislative cohesion. While it is true that the *value* of establishing a unique personal reputation grows with district magnitude in both electoral contexts, a representative must set this against (1) her diminished *ability* to distinguish herself from a larger group of co-partisans and (2) the electoral *costs* of doing so in the event that she is unsuccessful in securing enough preference votes. These costs are relatively higher in systems that are not completely flexible because of the party's ability to sanction dissent via list placement. While both the benefits *and* costs of dissent diminish in district magnitude, our results highlight that in both systems, these costs and benefits remain even at very high (indeed, arbitrarily high) levels of district magnitude. As a result, our framework is uniquely able to offer distinct hypotheses concerning district magnitude under flexible-list PR.

8. Additional Results, and Robustness

Incumbency Disadvantage. Our benchmark setting restricts the prior belief about each incumbent representative's alignment to $\mu \geq 1/2$. Proposition 1 characterizes a pure strategy equilibrium in which an aligned and a mis-aligned representative employ threshold strategies: each type of incumbent supports a policy if and only if the value of the policy exceeds a threshold. Proposition 3 highlights that there is an intermediate degree of list flexibility that maximizes the quality of dyadic representation.

When $\mu \geq 1/2$, the collective reputation of the incumbent team is bolstered when both incumbent representatives vote the same way—a split vote *harms* voters' perception of the team. Of course, *individual* reputation may improve, but there is always a cost to the party's reputation as a whole.

If $\mu < 1/2$, the collective reputation of the incumbent team may be *bolstered* when voters observe dissent among the co-partisan incumbents. While one representative's reputation is harmed by the split record, the other representative's reputation is improved to such an extent that it net *increases* voters' joint assessment of the representatives relative to the prior, when the party's reputation is initially weak.

Proposition 6. *Suppose $\mu < 1/2$. For κ not too small, there exists a symmetric equilibrium in mixed strategies, with thresholds $\underline{\theta}_0^{**}(\kappa)$, $\overline{\theta}_0^{**}(\kappa)$, $\underline{\theta}_b^{**}(\kappa)$ and $\underline{\theta}_b^{**}(\kappa)$, such that:*

1. *An aligned representative votes* nay *if $\theta \leq \underline{\theta}_0^{**}(\kappa)$, randomizes if $\theta \in (\underline{\theta}_0^{**}(\kappa), \overline{\theta}_0^{**}(\kappa))$, and votes* aye *if $\theta \geq \overline{\theta}_0^{**}(\kappa)$, and*

2. *An mis-aligned representative votes* nay *if $\theta \leq \underline{\theta}_b^{**}(\kappa)$, randomizes if $\theta \in (\underline{\theta}_b^{**}(\kappa), \overline{\theta}_b^{**}(\kappa))$, and votes* aye *if $\theta \geq \overline{\theta}_b^{**}(\kappa)$,*

There exists a threshold η^{**} such that a more flexible list worsens dyadic reputation if and only if $\eta \geq \eta^{**}$.

In the Appendix, we further show that when $\mu < \frac{1}{2}$, this is the unique symmetric equilibrium in which each representative's prospect of supporting the policy is an increasing function of θ.

Ballot Order Effects. In our benchmark setting, in between dates, the incumbent representatives may be strictly ordered according to their posterior reputation for constituency alignment. In that event, Lemma 3 shows that the leadership always places the incumbent with the lower reputation at the top of the ballot. It does so because (1) the order on the ballot has no effect on the total preference votes awarded to the party's candidates and (2) the leadership strictly values the reelection of a representative that is more likely to support its future initiatives.

The result that ballot order does not affect the total share of preference votes is valuable for our benchmark setting, since it highlights the mis-alignment between voters and party leaders over the promotion of candidates. There are, however, a number of directions in which the result can be relaxed. For example, under a non-uniform aggregate shock (ξ), the result is not generally true. In that context, Buisseret, Folke, Prato and Rickne (2017) develop and empirically test the hypothesis that seat-share maximizing parties may wish to place more popular candidates in ballot ranks where voters anticipate a higher relative prospect of making a difference—the "marginal rank" hypothesis.

Even in our uniform setting, rank-order effects may arise when $\mu_O \neq 1/2$. To see why, suppose that as in the body of the paper, $\mu \geq 1/2$ and that incumbent i voted *nay*, but representative j voted *aye*. This implies that i is aligned, and j is mis-aligned. If the party leadership places the mis-aligned incumbent at the top of the ballot, for general μ_O an uncommitted voter's relative value of casting a preference vote for aligned i instead of an opposition candidate (i.e., Equation 13) is now:

$$V_i(\mathsf{n},\mathsf{y}) - V_O|\{ji\} \propto \frac{1}{3}\underbrace{S(0-\mu_O)}_{\xi=\xi_1} + \frac{1}{3}\underbrace{S(1-0)}_{\xi=\xi_\eta} + \frac{1}{3}S\underbrace{(0-\mu_O)}_{\xi=\xi_2} = \frac{1}{3}S(1+0-2\mu_O). \tag{18}$$

If, instead, the party leader placed the more popular i at the top of the ballot, a voter's relative value of casting a preference vote for aligned i instead of an opposition candidate is:

$$V_i(\mathsf{n},\mathsf{y}) - V_O|\{ij\} \propto \frac{1}{2}\underbrace{S(1-\mu_O)}_{\xi=\xi_1} + \frac{1}{2}S\underbrace{(0-\mu_O)}_{\xi=\xi_2} = \frac{1}{2}S(1+0-2\mu_O). \tag{19}$$

When the Opposition party's candidates are favorably evaluated, i.e., $\mu_O > 1/2$, the Incumbent party leader's incentives to place the mis-aligned j at the top of the ballot are *augmented*, relative to our benchmark setting. The same incentive also arises in the symmetric equilibrium characterized in Proposition 6, i.e., when $\mu < 1/2$: the mean posterior reputation of the incumbents after a split vote lies above the prior μ, but below the Opposition candidates' reputation, μ_O. Thus, the leadership *strictly* prefers to place the mis-aligned candidate at the top of the ballot.

If, instead, the Opposition party's candidates are unfavorably evaluated, i.e., $\mu_O < 1/2$, the party leadership faces a trade-off between an electorally competitive ballot $\{ij\}$ and one that maximizes the prospect that—conditional on winning a single seat—its preferred candidate j is reelected, $\{ji\}$.

In the latter case, the party leadership faces a commitment problem: it could value a commitment to punish indisciplined representatives by demoting them to the bottom of the ballot, but face electoral incentives to instead promote these candidates to the very top. Interestingly, voters may also be worse off from the leader's inability to commit: the promise of promotion and preference votes unleashes incentives to obstruct to an even greater degree than in our benchmark setting, and the net consequence may be worse for dyadic representation.[27]

This further emphasizes the value of partially transferring electoral control from voters to party leaders via list assignments, despite the elite's total mis-alignment with voters over its most-preferred representative. Such a transfer can diminish personal vote-seeking (pandering) by legislators to the benefit of voters. But this mechanism may fail if leaders also face electoral incentives to promote obstructionist candidates—as argued, for example, by Crisp et al. (2013) in Slovakia.

Other Sources of Party Discipline. Our benchmark analysis endows party leaders with a single mechanism for enforcing discipline: the rank assignment on the party's list. In practice, however, parties have other ways to reward or punishment incumbent legislators for their (dis)loyalty. One especially relevant possibility is that the party might simply remove a representative from its list, altogether, replacing her in the event that she fails to vote with her party on critical bills.

A simple extension of our baseline model reveals that this is not a straightforward choice, however. To see why, suppose that a split record occurred at the first date and that all agents believe one representative (who voted against her party) is aligned and the other (who voted with her party) is mis-aligned, with probability one. One option for the party leadership is to punish the representative that voted against the party by removing her from the ballot, and replacing her with a fresh alternative. If the leadership does not replace the aligned representative, the team's average reputation for alignment is $\frac{1+0}{2}$. But if the leadership replaces the aligned representative with a fresh alternative, the team's average reputation for alignment falls to $\frac{\mu+0}{2}$. Intuitively, removing a very popular incumbent from the party's list carries an electoral cost: if the leadership is sufficiently concerned with maximizing electoral performance, i.e., its expected number of seats, the threat of removal may not be seen as credible. Notice that this cost does *not* apply in our benchmark setting when the leadership simply demotes a representative from the first to the second rank.

[27] Though not a feature of our model, in real-world settings, parties might also suffer reputation costs if their proposed order is overturned; for example, they may be seen as failing to correctly anticipate voters' preferences over candidates.

9. Conclusion

We develop a new framework to study legislative representation under open and flexible list proportional representation. We replace the classical *open* versus *closed* dichotomy of list PR systems with a continuum that fully reflects real-world variation in the extent to which preference votes versus list assignment primarily matters for an incumbent's prospect of reelection.

Our framework provides conditions under which an open list system generate worse dyadic representation than a closed list system, and also highlights how legislative cohesion under different degrees of flexibility—traditionally treated as equivalent to open list in existing work—may be lower than single-member contexts, or alternative be observationally equivalent to a closed system. These results stand in stark contrast to a conventional wisdom that trades off improved dyadic representation with cohesive legislative parties. They highlight circumstances under which both voters and parties stand to gain from a less flexible list, i.e., in which *principals do not compete*.

Scholars have argued that differences in electoral rules—in particular, majoritarian versus proportional—may account for cross-country variation in patterns of government formation, redistribution, party systems and turnout. Yet the majoritarian-proportional dichotomy misses tremendous cross-national variation in the operation of proportional rule systems. We hope that our results can inform future empirical scholarship, and encourage additional formal-theoretic work on these important electoral rules.

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Appendix

A. Equilibrium Analysis

Consider the problem facing a type τ representative $i \in \{A, B\}$, who learns the constituency value of the policy θ, and does not know the alignment of her co-partisan. Her expected relative value of voting in favor of the policy (i.e., voting y) is given by:[28]

$$R\,\mathbb{E}\left\{\begin{array}{l} l_i(\mathsf{y},\mathsf{t}_j)p_i(\mathsf{y},\mathsf{t}_j,\{ij\}) + (1-l_i(\mathsf{y},\mathsf{t}_j))p_i(\mathsf{y},\mathsf{t}_j,\{ji\}) \\ -l_i(\mathsf{n},\mathsf{t}_j)p_i(\mathsf{n},\mathsf{t}_j,\{ij\}) - (1-l_i(\mathsf{n},\mathsf{t}_j))p_i(\mathsf{n},\mathsf{t}_j,\{ji\}) \end{array}\right\} + \chi(\theta+\tau), \tag{20}$$

where the expectation is taken with respect to the strategy of the leader and the type and strategy of the other legislator, $p_i(\mathsf{t}_i, \mathsf{t}_j, l)$ is legislator i's winning probability as a function of the list assignment l—either $\{ij\}$ or $\{ji\}$—the vote tally $\mathsf{t} = (\mathsf{t}_i, \mathsf{t}_j)$, and where we use the shorthand $l_i(\mathsf{t}) = \mathbb{I}\{l(\mathsf{t}) = \{ij\}\}$.

Proof of Lemma 2. We derive $\max\{V_i, V_j\} - V_O$. Let $Q_{[x',x'']}(\cdot) \in [0,1]$ be the probability of passage of a policy agenda whose value is contained in an interval $[x', x'']$. When $\theta \in [0, b]$, there is no uncertainty about the behavior of an incumbent, so the only relevant belief is the one about the opposition challenger (μ_O); when $\theta \in [-b, 0]$, there is no uncertainty about the behavior of an opposition challenger, so the relevant beliefs are the ones about incumbents i and j. Recall our assumption that if an uncommitted voter is indifferent between casting her preference vote for a set of candidates, she randomizes uniformly over the subset of these candidates with the highest posterior reputation of alignment.

Fix a list $\{ij\}$. Using the voting calculus developed in expressions (2) and (3), we obtain the total preference votes accruing to the Incumbent ticket:

$$v_A + v_B = \alpha + (1-2\alpha)\left(\frac{1}{2} + \phi[\max\{V_A, V_B\} - V_O - \xi]\right),$$

so that the probability with which the Incumbent ticket wins both seats is:

$$\Pr(v_A + v_B \geq \pi) = \Pr\left(\xi \leq \max\{V_A, V_B\} - V_O - \frac{\pi - 1/2}{(1-2\alpha)\phi} \equiv \xi_2\right). \tag{21}$$

[28] For simplicity of exposition, we set aside the continuation values associated with second period expected policy outcomes. As the rest of the analysis clarifies, under our analytic approximation these continuation values will have no bearing on our results.

Likewise, the probability with which the Incumbent ticket wins a single seat is:

$$\Pr(\pi > v_A + v_B \geq 1-\pi) = \Pr\left(\xi_2 < \xi \leq \max\{V_A, V_B\} - V_O + \frac{\pi - 1/2}{(1-2\alpha)\phi} \equiv \xi_1\right). \tag{22}$$

Finally, conditional on winning a single seat, the prospect that second-ranked candidate j is elected is the prospect that $\frac{v_i}{v_i+v_j} \leq \eta$. This is impossible unless uncommitted voters that do not cast a preference vote for an Opposition candidate cast their preference votes for j. Thus, the prospect that second-ranked candidate j is elected is the prospect that $V_j > V_i$, or $V_j \geq V_i$ and $\hat{\mu}_j > \hat{\mu}_i$, and:

$$\Pr\left(\frac{v_i}{v_i+v_j} \leq \eta\right) = \Pr\left(\xi \geq \max\{V_A, V_B\} - V_O + \frac{1-\frac{\alpha}{\eta}}{2\phi(1-2\alpha)} \equiv \xi_\eta\right). \tag{23}$$

To ensure that $\xi_\eta \in (\xi_2, \xi_1)$, we need:

$$\xi_2 < \xi_\eta < \xi_1 \iff \frac{\alpha}{2\pi} \equiv \underline{\eta} < \eta < \frac{\alpha}{2(1-\pi)} \equiv \overline{\eta}. \tag{24}$$

For our benchmark analysis, we assume that $\eta \in (\underline{\eta}, \overline{\eta})$, which can be satisfied so long as $\alpha < 2(1-\pi)$, since $\alpha < 1/2 < \pi$.

Voters therefore evaluate that there are at most three decisive events. *First*, the incumbent ticket gets just enough votes to secure one seat that goes to the first-ranked incumbent i ($\xi = \xi_1$). *Second*, the incumbent ticket wins a single seat and the second-ranked candidate j gets just enough preference votes to achieve the quota ($\xi = \xi_\eta$). This happens if and only if enough uncommitted voters that support an incumbent candidate support the second-ranked candidate. *Third*, the incumbent ticket gets just enough votes to secure two seats ($\xi = \xi_2$).

We therefore define:

$$\Pr(\text{pivot}) \equiv \Pr(\xi = \xi_2) + \Pr(\xi = \xi_\eta) + \Pr(\xi = \xi_1),$$

and conclude that V_i can be written as:

$$\begin{aligned} V_i = & \frac{\Pr(\xi = \xi_2)}{\Pr(\text{pivot})} \left\{ \begin{array}{c} (\hat{\mu}_i(\mathsf{t}) + \hat{\mu}_j(\mathsf{t}))S + \mathbb{E}[\theta \leq -b]q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \geq b]q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \leq \theta \leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}), \hat{\mu}_j(\mathsf{t})) + \mathbb{E}[0 \leq \theta \leq b]q(\mathsf{y},\mathsf{y}) \end{array} \right\} \\ & + \frac{\Pr(\xi = \xi_\eta)}{\Pr(\text{pivot})} \left\{ \begin{array}{c} (\hat{\mu}_i(\mathsf{t}) + \mu_O)S + \mathbb{E}[\theta \leq -b]q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \geq b]q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \leq \theta \leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t})) + \mathbb{E}[0 \leq \theta \leq b]Q_{[0,b]}(\mu_O) \end{array} \right\} \end{aligned}$$

$$+\frac{\Pr(\xi=\xi_1)}{\Pr(\text{pivot})}\left\{\begin{array}{c}(\hat{\mu}_i(\mathsf{t})+\mu_O)S+\mathbb{E}[\theta\leq -b]q(\mathsf{n},\mathsf{n})+\mathbb{E}[\theta\geq b]q(\mathsf{y},\mathsf{y})\\ +\mathbb{E}[-b\leq\theta\leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}))+\mathbb{E}[0\leq\theta\leq b]Q_{[0,b]}(\mu_O)\end{array}\right\}$$

where $Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}),\hat{\mu}_j(\mathsf{t}))$ is the probability that a policy with value $\theta\in[-b,0]$ passes given that the two representatives are from the Incumbent party, and each is believed to be aligned with probability $\hat{\mu}_i(\mathsf{t})$ and $\hat{\mu}_j(\mathsf{t})$, where t is the first-period vote tally, $Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}))$ is the probability that a policy with value $\theta\in[-b,0]$ passes given that one representative is in the Opposition party (and thus votes against the policy regardless of alignment), while the other is from the Incumbent party and is aligned with probability $\hat{\mu}_i(\mathsf{t})$, and $Q_{[0,b]}(\mu_O)$ is the probability that a policy with value $\theta\in[0,b]$ passes given that one representative is in the Opposition party and is aligned with probability μ_O, while the other is from the Incumbent party and thus votes for the policy regardless of alignment.

Similarly, we have

$$\begin{aligned}V_j=&\frac{\Pr(\xi=\xi_2)}{\Pr(\text{pivot})}\left\{\begin{array}{c}(\hat{\mu}_i(\mathsf{t})+\hat{\mu}_j(\mathsf{t}))S+\mathbb{E}[\theta\leq -b]q(\mathsf{n},\mathsf{n})+\mathbb{E}[\theta\geq b]q(\mathsf{y},\mathsf{y})\\ +\mathbb{E}[-b\leq\theta\leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}),\hat{\mu}_j(\mathsf{t}))+\mathbb{E}[0\leq\theta\leq b]q(\mathsf{y},\mathsf{y})\end{array}\right\}\\ &+\frac{\Pr(\xi=\xi_\eta)}{\Pr(\text{pivot})}\left\{\begin{array}{c}(\hat{\mu}_j(\mathsf{t})+\mu_O)S+\mathbb{E}[\theta\leq -b]q(\mathsf{n},\mathsf{n})+\mathbb{E}[\theta\geq b]q(\mathsf{y},\mathsf{y})\\ +\mathbb{E}[-b\leq\theta\leq 0]Q_{[-b,0]}(\hat{\mu}_j(\mathsf{t}))+\mathbb{E}[0\leq\theta\leq b]Q_{[0,b]}(\mu_O)\end{array}\right\}\\ &+\frac{\Pr(\xi=\xi_1)}{\Pr(\text{pivot})}\left\{\begin{array}{c}(\hat{\mu}_i(\mathsf{t})+\mu_O)S+\mathbb{E}[\theta\leq -b]q(\mathsf{n},\mathsf{n})+\mathbb{E}[\theta\geq b]q(\mathsf{y},\mathsf{y})\\ +\mathbb{E}[-b\leq\theta\leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}))+\mathbb{E}[0\leq\theta\leq b]Q_{[0,b]}(\mu_O)\end{array}\right\}\end{aligned}$$

and

$$\begin{aligned}V_O=&\frac{\Pr(\xi=\xi_2)}{\Pr(\text{pivot})}\left\{\begin{array}{c}(\hat{\mu}_i(\mathsf{t})\mathbb{I}_{\{V_i\geq V_J\}}+\hat{\mu}_j(\mathsf{t})\mathbb{I}_{\{V_i<V_J\}}+\mu_O)S+\mathbb{E}[\theta\leq -b]q(\mathsf{n},\mathsf{n})+\mathbb{E}[\theta\geq b]q(\mathsf{y},\mathsf{y})\\ +\mathbb{E}[-b\leq\theta\leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}))+\mathbb{E}[0\leq\theta\leq b]Q_{[0,b]}(\mu_O)\end{array}\right\}\\ &+\frac{\Pr(\xi=\xi_\eta)}{\Pr(\text{pivot})}\left\{\begin{array}{c}(\hat{\mu}_i(\mathsf{t})+\mu_O)S+\mathbb{E}[\theta\leq -b]q(\mathsf{n},\mathsf{n})+\mathbb{E}[\theta\geq b]q(\mathsf{y},\mathsf{y})\\ +\mathbb{E}[-b\leq\theta\leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}))+\mathbb{E}[0\leq\theta\leq b]Q_{[0,b]}(\mu_O)\end{array}\right\}\\ &+\frac{\Pr(\xi=\xi_1)}{\Pr(\text{pivot})}\left\{\begin{array}{c}2\mu_O S+\mathbb{E}[\theta\leq -b]q(\mathsf{n},\mathsf{n})+\mathbb{E}[\theta\geq b]q(\mathsf{y},\mathsf{y})\\ +\mathbb{E}[-b\leq\theta\leq 0]q(\mathsf{n},\mathsf{n})+\mathbb{E}[0\leq\theta\leq b]Q_{[0,b]}(\mu_O,\mu_O)\end{array}\right\}\end{aligned}$$

Lemma A.1. *Given a list $\{ij\}$: if $\hat{\mu}_i\geq\hat{\mu}_j$, $V_i\geq V_j$, and if $\hat{\mu}_i<\hat{\mu}_j$, $V_i<V_j$.*

Proof. The first part is by inspection. Consider the second part: suppose $\hat{\mu}_i<\hat{\mu}_j$ and $V_i\geq V_j$. Then, $\Pr(\xi=\xi_\eta)=0$, which implies $V_i=V_j$. Since the incumbent candidates are strictly ordered by reputation, an uncommitted voter anticipates that other uncommitted voters who do not support an Opposition candidate resolve their indifference over Incumbent candidates in favor of j. But this implies $\Pr(\xi=\xi_\eta)>0$, and thus $V_j>V_i$. □

We conclude that $V_i \geq V_j$ if and only if $\hat{\mu}_i \geq \hat{\mu}_j$. As κ approaches infinity, we have that $\mathbb{E}[-b \leq \theta \leq 0]$ and $\mathbb{E}[0 \leq \theta \leq b]$ approach zero (and thus can be written as $\mathcal{O}(1/\kappa)$). Moreover, since $\xi \sim U$, $V_i < V_j$ implies $\frac{\Pr(\xi=\xi_2)}{\Pr(\text{pivot})} = \frac{\Pr(\xi=\xi_\eta)}{\Pr(\text{pivot})} = \frac{\Pr(\xi=\xi_1)}{\Pr(\text{pivot})} = \frac{1}{3}$ and $V_i \geq V_j$ implies $\frac{\Pr(\xi=\xi_2)}{\Pr(\text{pivot})} = \frac{\Pr(\xi=\xi_1)}{\Pr(\text{pivot})} = \frac{1}{2}$.

Hence, if the list is $\{ij\}$ and $\hat{\mu}_i \geq \hat{\mu}_j$, we obtain:

$$[\max\{V_i, V_j\} - V_O]_{\{ij\}} = [V_i - V_O]_{\{ij\}} = \frac{S}{2}\big(\hat{\mu}_i(\mathsf{t}) + \hat{\mu}_j(\mathsf{t}) - 1\big) + \mathcal{O}(1/\kappa). \tag{25}$$

If the list is $\{ij\}$ and, instead, $\hat{\mu}_j > \hat{\mu}_i$ we obtain:

$$[\max\{V_i, V_j\} - V_O]_{\{ij\}} = [V_j - V_O]_{\{ij\}} = \frac{S}{3}\big(\hat{\mu}_i(\mathsf{t}) + \hat{\mu}_j(\mathsf{t}) - 1\big) + \mathcal{O}(1/\kappa). \tag{26}$$

We next establish properties of voters' posterior beliefs about politicians after a vote tally $(\mathsf{t}_A, \mathsf{t}_B) = (\mathsf{y}, \mathsf{y})$ and $(\mathsf{t}_A, \mathsf{t}_B) = (\mathsf{n}, \mathsf{n})$, in any symmetric equilibrium. Let $\mathcal{L}$ denote the set of mixed strategies $l = (l_0(\theta), l_b(\theta))$ for each pair of types, where $l_\tau(\theta)$ is the probability that a type-τ representative votes in favor of the policy when the constituency value is $\theta \in [-\kappa, \kappa]$:

$$\mathcal{L} = \Big\{ l : [-\kappa, \kappa] \rightrightarrows [0,1]^2 \Big\}$$

Letting $l_\mu(\theta) \equiv (1-\mu) l_b(\theta) + \mu l_0(\theta)$, posterior beliefs after a vote tally $(\mathsf{t}_A, \mathsf{t}_B) = (y, y)$ and $(\mathsf{t}_A, \mathsf{t}_B) = (n, n)$ are:

$$\mu(\mathsf{y}, \mathsf{y}) = \frac{\mu \int_{-\kappa}^{\kappa} l_0(z) l_\mu(z) dz}{\mu \int_{-\kappa}^{\kappa} l_0(z) l_\mu(z) dz + (1-\mu) \int_{-\kappa}^{\kappa} l_b(z) l_\mu(z) dz} \tag{27}$$

$$\mu(\mathsf{n}, \mathsf{n}) = \frac{\mu \int_{-\kappa}^{\kappa} (1 - l_0(z))(1 - l_\mu(z)) dz}{\mu \int_{-\kappa}^{\kappa} (1 - l_0(z))(1 - l_\mu(z)) dz + (1-\mu) \int_{-\kappa}^{\kappa} (1 - l_b(z))(1 - l_\mu(z)) dz}. \tag{28}$$

Expression (20) implies that, in any symmetric equilibrium, $l_0 \in (0,1] \Rightarrow l_b = 1$ and $l_b \in [0,1) \Rightarrow l_0 = 0$, and thus $\hat{\mu}(\mathsf{y}, \mathsf{y}) < \mu < \hat{\mu}(\mathsf{n}, \mathsf{n})$. Next, define:

$$\theta^l = -\frac{RS\psi}{\chi} - \frac{3}{2} + \frac{1}{2\phi S} \frac{\eta - \alpha}{\eta(1-2\alpha)}. \tag{29}$$

$$\theta^h = \frac{RS\psi}{\chi} + \frac{1}{2} + \frac{1}{2\phi S} \frac{\eta - \alpha}{\eta(1-2\alpha)}. \tag{30}$$

We observe that (i) neither of these thresholds depends on κ, and (ii) whenever $\theta < \theta^l$, each type strictly prefers $l_\tau = 0$, and (iii) whenever $\theta > \theta^h$, each type strictly prefers $l_\tau = 1$. Defining the space:

$$\mathcal{L}^* = \left\{ l \in \mathcal{L} \;\middle|\; \begin{array}{c} l_\tau(\theta) = 0 \quad \forall \theta \leq \theta^l, \tau \in \{0, b\} \\ l_\tau(\theta) = 1 \quad \forall \theta \geq \theta^h, \tau \in \{0, b\} \\ l_0(\theta) \in (0,1] \Rightarrow l_b(\theta) = 1, \; l_b(\theta) \in [0,1) \Rightarrow l_0(\theta) = 0 \end{array} \right\}$$

we conclude that a symmetric equilibrium strategy is an element of $\mathcal{L}^*$. We further argue that for any $l \in \mathcal{L}^*$, $\lim_{\kappa\to\infty} \hat{\mu}(\mathsf{y},\mathsf{y}) = \mu = \lim_{\kappa\to\infty} \hat{\mu}(\mathsf{n},\mathsf{n}))$. We prove the first limit, since the argument for $\mu(\mathsf{n},\mathsf{n})$ is symmetric. By definition of $\mathcal{L}^*$, we have:

$$\frac{1 - F(\theta^h)}{1 - F(\theta^l)} \leq \frac{\int_{-\kappa}^{\kappa} l_b(z) l_\mu(z) dz}{\int_{-\kappa}^{\kappa} l_0(z) l_\mu(z) dz} \leq \frac{1 - F(\theta^l)}{1 - F(\theta^h)}. \tag{31}$$

Since:

$$\frac{\lim_{\kappa\to\infty}(1 - F(\theta^h))}{\lim_{\kappa\to\infty}(1 - F(\theta^l))} = \frac{\lim_{\kappa\to\infty}(1 - \frac{\theta^h}{2\kappa})}{\lim_{\kappa\to\infty}(1 - \frac{\theta^l}{2\kappa})} = 1 = \frac{\lim_{\kappa\to\infty}(1 - \frac{\theta^l}{2\kappa})}{\lim_{\kappa\to\infty}(1 - \frac{\theta^h}{2\kappa})} = \frac{\lim_{\kappa\to\infty}(1 - F(\theta^l))}{\lim_{\kappa\to\infty}(1 - F(\theta^h))}, \tag{32}$$

we conclude that for any $l \in \mathcal{L}^*$:

$$\lim_{\kappa\to\infty} \hat{\mu}(\mathsf{y},\mathsf{y}) = \frac{\mu}{\mu + (1-\mu)\frac{\lim_{\kappa\to\infty} \int_{-\kappa}^{\kappa} l_b(z) l_\mu(z) dz}{\lim_{\kappa\to\infty} \int_{-\kappa}^{\kappa} l_0(z) l_\mu(z) dz}} = \mu. \tag{33}$$

It follows that if $l \in \mathcal{L}^*$, $\lim_{\kappa\to\infty} \hat{\mu}(\mathsf{y},\mathsf{y}) = \mu = \lim_{\kappa\to\infty} \hat{\mu}(\mathsf{n},\mathsf{n})$. Finally, we have that $\hat{\mu}_i(\mathsf{y}_i,\mathsf{n}_j) = \mu \frac{\int_{-\kappa}^{\kappa} l_0(z)(1-l_\mu(z))\,dz}{\int_{-\kappa}^{\kappa} l_\mu(z)(1-l_\mu(z))\,dz}$ and $\hat{\mu}_j(\mathsf{y}_i,\mathsf{n}_j) = \mu \frac{\int_{-\kappa}^{\kappa} (1-l_0(z)) l_\mu(z)\,dz}{\int_{-\kappa}^{\kappa} l_\mu(z)(1-l_\mu(z)\,dz}$. We will subsequently show that in the symmetric equilibria we study, neither of these posterior beliefs is a function of κ. Taking (what we have shown to be well-defined) limits on expressions (25) and (26), and recalling that $\mu_O = 1/2$, we conclude that if the list is $\{ij\}$ and $\hat{\mu}_i \geq \hat{\mu}_j$, we obtain:

$$\lim_{\kappa\to\infty} [\max\{V_i, V_j\} - V_O] = \lim_{\kappa\to\infty} [V_i - V_O] = \frac{S}{2} \lim_{\kappa\to\infty} \big(\hat{\mu}_i(\mathsf{t}) + \hat{\mu}_j(\mathsf{t}) - 1\big).$$

If the list is $\{ij\}$ and, instead, $\hat{\mu}_j > \hat{\mu}_i$ we obtain:

$$\lim_{\kappa\to\infty} [\max\{V_i, V_j\} - V_O] = \lim_{\kappa\to\infty} [V_j - V_O] = \frac{S}{3} \lim_{\kappa\to\infty} \big(\hat{\mu}_i(\mathsf{t}) + \hat{\mu}_j(\mathsf{t}) - 1\big). \quad \square$$

Proof of Lemma 3. Let $\hat{\mu}_i(\mathsf{n}_i,\mathsf{y}_j)$ denote the posterior reputation of representative i after she votes 'no' and representative j votes 'yes', and let $\hat{\mu}_j(\mathsf{n}_i,\mathsf{y}_j)$ denote the posterior reputation of representative j after she votes 'yes' and representative i votes 'no'. Then,

$$\mu_{\text{split}}(l) \equiv \frac{\hat{\mu}_i(\mathsf{y}_i,\mathsf{n}_j) + \hat{\mu}_j(\mathsf{y}_i,\mathsf{n}_j)}{2}$$

denotes the mean posterior reputation of the Incumbent ticket after a split vote, where we emphasize the dependence on the strategy $l \in \mathcal{L}^*$. We now prove a more general result.

Lemma 3* If $\mu_{\text{split}}(l) \leq 1/2$, and $\hat{\mu}_i(\mathsf{t}) > \hat{\mu}_j(\mathsf{t})$, then the leadership strictly prefers the list assignment $\{ji\}$.

Proof. By Lemma 2, $\hat{\mu}_i > \hat{\mu}_j$ Lemma A.1 implies $V_i \geq V_j$ regardless of the choice $\{ij\}$ or $\{ji\}$. Thus uncommitted voters that do not support an Opposition candidate cast a preference vote for representative i. Moreover, expressions (25) and (26) highlight that when $\mu_{\text{split}}(l) \leq \frac{1}{2}$ and $\hat{\mu}_i(\mathsf{t}) > \hat{\mu}_j(\mathsf{t})$, for κ sufficiently large, $[\max\{V_i, V_j\} - V_O]_{\{ji\}} \geq [\max\{V_i, V_j\} - V_O]_{\{ij\}}$. This implies that, relative to the list order $\{ij\}$, the list order $\{ji\}$ (weakly) increases both ξ_2 and ξ_1 (defined in expressions (21) and (22)), while $\xi_2 - \xi_1$ remains constant. Thus, it is sufficient to observe that since $\eta < \frac{\alpha}{2(1-\pi)}$, the probability that representative j is elected conditional on $\xi \in (\xi_2, \xi_1)$ is strictly higher under the list assignment $\{ji\}$ than under the list assignment $\{ij\}$. □
Lemma 3 is then a special case of Lemma 3* evaluated at $\mu_{\text{split}}(l) = \frac{1}{2}$. □

Proof of Proposition 1. Consider the problem of politician i. Recall that $p_i(\mathsf{t}_i, \mathsf{t}_j, k)$ for $k = \{ij\}$ or $k = \{ji\}$ denotes representative i's probability of reelection when the first-period vote tally is $(\mathsf{t}_i, \mathsf{t}_j)$, and the list assignment is k, and $\hat{\mu}_i(\mathsf{t}, \mathsf{t}_j)$ is the posterior belief about representative i when her first period vote is $\mathsf{t} \in \{\mathsf{y}, \mathsf{n}\}$, and her co-partisan incumbent j's first period vote is $\mathsf{t}_j \in \{y, n\}$. We define:

$$\delta_p(\mathsf{y}; \kappa) = \frac{1}{2}\Big(p_i(\mathsf{y}, \mathsf{y}, \{ij\}) + p_i(\mathsf{y}, \mathsf{y}, \{ji\})\Big) - p_i(\mathsf{n}, \mathsf{y}, \{ji\})$$
$$\delta_p(\mathsf{n}; \kappa) = p_i(\mathsf{y}, \mathsf{n}, \{ij\}) - \frac{1}{2}\Big(p_i(\mathsf{n}, \mathsf{n}, \{ij\}) + p_i(\mathsf{n}, \mathsf{n}, \{ji\})\Big)$$

That is, $\delta_p(\mathsf{y})$ is the relative change in i's prospect of reelection from voting y when representative j votes y, while $\delta_p(\mathsf{n})$ is the relative change in i's prospect of reelection from voting y when representative j votes n.

Using the probabilities of winning and the expressions for $\max\{V_A(\mathsf{t}), V_B(\mathsf{t})\} - V_O$ in the proof of Lemma 2, and

$$\zeta(\alpha, \eta, \phi) = \frac{1}{2\phi S}\frac{\eta - \alpha}{\eta(1 - 2\alpha)},$$

we have:

$$p_i(\mathsf{y}, \mathsf{y}, \{ij\}) + p_i(\mathsf{y}, \mathsf{y}, \{ji\}) = 1 + 2\psi S\big(\hat{\mu}(\mathsf{y}, \mathsf{y}) - \mu_O\big) + \mathcal{O}(1/\kappa) \quad (34)$$
$$p_i(\mathsf{n}, \mathsf{y}, \{ji\}) = \frac{1}{2} + \frac{2}{3}\psi S(\mu_{\text{split}}(l) - \mu_O) + \psi S\zeta(\alpha, \gamma, \phi) + \mathcal{O}(1/\kappa) \quad (35)$$
$$p_i(\mathsf{y}, \mathsf{n}, \{ij\}) = \frac{1}{2} + \frac{2}{3}\psi S(\mu_{\text{split}}(l) - \mu_O) - \psi S\zeta(\alpha, \gamma, \phi) + \mathcal{O}(1/\kappa) \quad (36)$$

$$p_i(\mathsf{n},\mathsf{n},\{ij\}) + p_i(\mathsf{n},\mathsf{n},\{ji\}) = 1 + 2\psi S\big(\hat{\mu}(\mathsf{n},\mathsf{n}) - \mu_O\big) + \mathcal{O}(1/\kappa). \tag{37}$$

For finite κ, we can substitute equations (34)-(37) to obtain

$$\delta_p(\mathsf{y};\kappa) = \psi S\big(\hat{\mu}(\mathsf{y},\mathsf{y}) - \mu_O\big) - \frac{2}{3}\psi S\big(\mu_{\text{split}}(l) - \mu_O\big) - \psi S\zeta(\alpha,\gamma,\phi) + \mathcal{O}(1/\kappa) \tag{38}$$
$$\delta_p(\mathsf{n};\kappa) = \psi S\big(\mu_O - \hat{\mu}(\mathsf{n},\mathsf{n})\big) + \frac{2}{3}\psi S\big(\mu_{\text{split}}(l) - \mu_O\big) - \psi S\zeta(\alpha,\gamma,\phi) + \mathcal{O}(1/\kappa). \tag{39}$$

Having shown that in any symmetric equilibrium, $\lim_{\kappa\to\infty}\hat{\mu}(\mathsf{y},\mathsf{y}) = \lim_{\kappa\to\infty}\hat{\mu}(\mathsf{y},\mathsf{y}) = \mu$, and recalling that $\mu_O = 1/2$, we therefore have that for κ large enough:

$$\begin{aligned}\mu_{\text{split}}(l) < (3/2)\mu - 1/4 \Rightarrow \delta_p(\mathsf{y};\kappa) - \delta_p(\mathsf{n};\kappa) > 0,\\ \mu_{\text{split}}(l) > (3/2)\mu - 1/4 \Rightarrow \delta_p(\mathsf{y};\kappa) - \delta_p(\mathsf{n};\kappa) < 0.\end{aligned} \tag{40}$$

We let $D_\tau(\theta; l(\theta))$ denote the net value of supporting the policy agenda by a representative of type τ when the constituency value is θ, in a symmetric equilibrium where the strategies of the types are $l(\theta) = (l_0(\theta), l_b(\theta)) \in \mathcal{L}^*$, i.e.,

$$D_\tau(\theta; l(\theta)) = R\left[l_\mu(\theta)\delta_p(\mathsf{y};\kappa) + (1 - l_\mu(\theta))\delta_p(\mathsf{n};\kappa)\right] + \chi(\theta + \tau).$$

For a strategy $l \in \mathcal{L}^*$, we may define the following thresholds $\{\hat{\theta}_\tau(\kappa), \check{\theta}_\tau(\kappa)\}_{\tau\in\{0,b\}}$, implicitly:

$$D_0(\hat{\theta}_0(\kappa); l) = R(1-\mu)\delta_p(\mathsf{y};\kappa) + R\mu\delta_p(\mathsf{n};\kappa) + \chi\hat{\theta}_0(\kappa) = 0 \tag{41}$$
$$D_0(\check{\theta}_0(\kappa); l) = R\delta_p(\mathsf{y};\kappa) + \chi\check{\theta}_0(\kappa) = 0 \tag{42}$$
$$D_b(\hat{\theta}_b(\kappa); l) = R\delta_p(\mathsf{n};\kappa) + \chi(\hat{\theta}_b(\kappa) + b) = 0 \tag{43}$$
$$D_b(\check{\theta}_b(\kappa); l) = R(1-\mu)\delta_p(\mathsf{y};\kappa) + R\mu\delta_p(\mathsf{n};\kappa) + \chi(\check{\theta}_b(\kappa) + b) = 0. \tag{44}$$

Note that if $-\kappa < \theta^l$ and $\kappa > \theta^h$—which we henceforth impose—these thresholds are always contained in $[-\kappa, \kappa]$.

We proceed to establish the following equilibrium characterization.

Lemma A.2. *For κ sufficiently large, the following strategy profile constitutes a symmetric equilibrium:*

$$l_0^*(\theta) = \begin{cases} 0 & \theta < \min\{\hat{\theta}_0(\kappa), \check{\theta}_0(\kappa)\} \\ \mathbf{I}_{\{\mu\geq 1/2\}} + \mathbf{I}_{\{\mu<1/2\}}\frac{\theta - \hat{\theta}_0(\kappa)}{\check{\theta}_0(\kappa) - \hat{\theta}_0(\kappa)} & \theta \in \Big[\min\{\hat{\theta}_0(\kappa), \check{\theta}_0(\kappa)\}, \max\{\hat{\theta}_0(\kappa), \check{\theta}_0(\kappa)\}\Big) \\ 1 & \theta \geq \max\{\hat{\theta}_0(\kappa), \check{\theta}_0(\kappa)\} \end{cases}$$

$$l_b^*(\theta) = \begin{cases} 0 & \theta < \min\{\hat{\theta}_b(\kappa), \check{\theta}_b(\kappa)\} \\ \mathbf{I}_{\{\mu<1/2\}} \frac{\theta - \hat{\theta}_b(\kappa)}{\check{\theta}_b(\kappa) - \hat{\theta}_b(\kappa)} & \theta \in \Big[\min\{\hat{\theta}_b(\kappa), \check{\theta}_b(\kappa)\}, \max\{\hat{\theta}_b(\kappa), \check{\theta}_b(\kappa)\}\Big] \\ 1 & \theta \geq \max\{\hat{\theta}_b(\kappa), \check{\theta}_b(\kappa)\} \end{cases}$$

Proof. We proceed by separately analyzing the cases $\mu \geq \frac{1}{2}$ and $\mu < \frac{1}{2}$.

Case 1: $\mu \geq \frac{1}{2}$. Under the strategy profile, $\mu_{\text{split}}(l) = \frac{1}{2}$. By inspection of (40), there exists a finite κ above which, under this strategy profile, $\delta_p(\mathsf{y}; \kappa) \geq \delta_p(\mathsf{n}; \kappa)$. Existence is easily verified by noting that $\delta_p(\mathsf{y}; \kappa) \geq \delta_p(\mathsf{n}; \kappa)$ implies $\hat{\theta}_b(\kappa) \geq \check{\theta}_b(\kappa)$, $\hat{\theta}_0(\kappa) \geq \check{\theta}_0(\kappa)$, and $\check{\theta}_b(\kappa) < \hat{\theta}_0(\kappa)$.

Case 2: $\mu < \frac{1}{2}$. We first show that, under the strategy profile, $\mu_{\text{split}}(l) \in (\mu, 1/2)$. By inspection of (40), $\mu_{\text{split}}(l) > \mu$ implies that for κ sufficiently large, $\delta_p(\mathsf{y}; \kappa) < \delta_p(\mathsf{n}; \kappa)$, which in turn yields $\hat{\theta}_b(\kappa) < \check{\theta}_b(\kappa) < \hat{\theta}_0(\kappa) < \check{\theta}_0(\kappa)$. And, we showed in Lemma 3* that $\mu_{\text{split}}(l) < 1/2$ implies that the leadership strictly prefers the ballot order $\{ji\}$ after a split vote tally $(\mathsf{n}_i, \mathsf{y}_j)$. To verify $\mu_{\text{split}}(l) \in (\mu, 1/2)$, observe that:

$$\mu_i(\mathsf{n}_i, \mathsf{y}_j) + \mu_j(\mathsf{n}_i, \mathsf{y}_j) = \mu \frac{\int_{-\kappa}^{\kappa} l_0(z)(1 - l_\mu(z)) + l_\mu(z)(1 - l_0(z))dz}{\int_{-\kappa}^{\kappa} l_\mu(z)(1 - l_\mu(z))dz}.$$

Under the strategy profile, we therefore have:

$$\mu_i(\mathsf{n}, \mathsf{y}) + \mu_j(\mathsf{n}, \mathsf{y})$$
$$= \mu \frac{(1-\mu)\Big[\int_{\hat{\theta}_b(\kappa)}^{\check{\theta}_b(\kappa)} l_b(z)dz + \int_{\check{\theta}_b(\kappa)}^{\hat{\theta}_0(\kappa)} dz\Big] + \int_{\hat{\theta}_0(\kappa)}^{\check{\theta}_0(\kappa)} (1 - l_0)(1 - \mu(1 - 2l_0))dz}{(1-\mu)\Big[\int_{\hat{\theta}_b(\kappa)}^{\check{\theta}_b(\kappa)} l_b(z)(1 - (1-\mu)l_b)dz + \mu \int_{\check{\theta}_b(\kappa)}^{\hat{\theta}_0(\kappa)} dz\Big] + \int_{\hat{\theta}_0(\kappa)}^{\check{\theta}_0(\kappa)} (1 - l_0)\mu(1 - \mu(1 - l_0))dz}. \tag{45}$$

We verify that $\mu_{\text{split}}(l) > \mu$, leaving the second result that $\mu_{\text{split}}(l) < 1/2$ to the reader. We have:

$$\mu_i(\mathsf{n}, \mathsf{y}) + \mu_j(\mathsf{n}, \mathsf{y}) - 2\mu \propto \tag{46}$$
$$(1-\mu)\Big[\int_{\hat{\theta}_b}^{\check{\theta}_b} l_b(z)dz + \int_{\check{\theta}_b}^{\hat{\theta}_0} dz\Big] + \int_{\hat{\theta}_0}^{\check{\theta}_0} (1 - l_0)(1 - \mu(1 - 2l_0))dz + \tag{47}$$
$$-2(1-\mu)\Big[\int_{\hat{\theta}_b}^{\check{\theta}_b} l_b(z)(1 - (1-\mu)l_b)dz + \mu \int_{\check{\theta}_b}^{\hat{\theta}_0} dz\Big] - 2\int_{\hat{\theta}_0}^{\check{\theta}_0} (1 - l_0)\mu(1 - \mu(1 - l_0))dz \tag{48}$$
$$= (1-\mu)\int_{\hat{\theta}_b}^{\check{\theta}_b} \Big[l_b - 2l_b + 2l_b^2(1-\mu)\Big]dz + (1-\mu)\int_{\check{\theta}_b}^{\hat{\theta}_0} (1 - 2\mu)dz \tag{49}$$
$$+ \int_{\hat{\theta}_0}^{\check{\theta}_0} (1 - l_0)\Big[1 - \mu(1 - 2l_0) - 2\mu + 2\mu^2(1 - l_0)\Big]dz \tag{50}$$
$$= (1-\mu)\int_{\hat{\theta}_b}^{\check{\theta}_b} l_b\Big[2l_b(1-\mu) - 1\Big]dz + (1-\mu)(\hat{\theta}_0 - \check{\theta}_b)(1 - 2\mu) \tag{51}$$

$$+\int_{\hat{\theta}_0}^{\check{\theta}_0}(1-l_0)(1-\mu)\Big[1-2\mu(1-l_0)\Big]dz \tag{52}$$

$$\propto\int_{\hat{\theta}_b}^{\check{\theta}_b} l_b\Big[2l_b(1-\mu)-1\Big]dz+(\hat{\theta}_0-\check{\theta}_b)(1-2\mu)+\int_{\hat{\theta}_0}^{\check{\theta}_0}(1-l_0)\Big[1-2\mu(1-l_0)\Big]dz \tag{53}$$

$$=(\hat{\theta}_0-\check{\theta}_b)(1-2\mu)-\int_{\hat{\theta}_b}^{\check{\theta}_b} l_b dz+2(1-\mu)\int_{\hat{\theta}_b}^{\check{\theta}_b} l_b^2 dz+\int_{\hat{\theta}_0}^{\check{\theta}_0}(1-l_0)dz-2\mu\int_{\hat{\theta}_0}^{\check{\theta}_0}(1-l_0)^2 dz \tag{54}$$

$$=(\hat{\theta}_0-\check{\theta}_b)(1-2\mu)-\int_{\hat{\theta}_b}^{\check{\theta}_b}\frac{z-\hat{\theta}_b}{\check{\theta}_b-\hat{\theta}_b}dz+2(1-\mu)\int_{\hat{\theta}_b}^{\check{\theta}_b}\left(\frac{z-\hat{\theta}_b}{\check{\theta}_b-\hat{\theta}_b}\right)^2 dz \tag{55}$$

$$+\int_{\hat{\theta}_0}^{\check{\theta}_0}\frac{\check{\theta}_0-z}{\check{\theta}_0-\hat{\theta}_0}dz-2\mu\int_{\hat{\theta}_0}^{\check{\theta}_0}\left(\frac{\check{\theta}_0-z}{\check{\theta}_0-\hat{\theta}_0}\right)^2 dz \tag{56}$$

$$=(\hat{\theta}_0-\check{\theta}_b)(1-2\mu)-\frac{\check{\theta}_b-\hat{\theta}_b}{2}+2(1-\mu)\frac{\check{\theta}_b-\hat{\theta}_b}{3}+\frac{\check{\theta}_0-\hat{\theta}_0}{2}-2\mu\frac{\check{\theta}_0-\hat{\theta}_0}{3}. \tag{57}$$

Simple algebra yields that

$$\begin{aligned}
\hat{\theta}_b(\kappa) &= -\frac{R}{\chi}\delta_p(\mathsf{n};\kappa)-b\\
\check{\theta}_b(\kappa) &= -\frac{R}{\chi}\Big[\mu\delta_p(\mathsf{n};\kappa)+(1-\mu)\delta_p(\mathsf{y};\kappa)\Big]-b\\
\hat{\theta}_0(\kappa) &= -\frac{R}{\chi}\Big[\mu\delta_p(\mathsf{n};\kappa)+(1-\mu)\delta_p(\mathsf{y};\kappa)\Big]\\
\check{\theta}_0(\kappa) &= -\frac{R}{\chi}\delta_p(\mathsf{y};\kappa).
\end{aligned}$$

Further recalling that $l_\tau(z)=\frac{\theta-\hat{\theta}_\tau(\kappa)}{\check{\theta}_\tau(\kappa)-\hat{\theta}_\tau(\kappa)}$ for $\tau\in\{b,0\}$. Substituting these thresholds and strategies into (57) yields:

$$\mu_i(\mathsf{n},\mathsf{y})+\mu_j(\mathsf{n},\mathsf{y})-2\mu\propto b(1-2\mu)+\frac{R}{\chi}\Big[\delta_p(\mathsf{n};\kappa)-\delta_p(\mathsf{y};\kappa)\Big]\frac{1-2\mu}{6}. \tag{58}$$

Since $\mu_i(\mathsf{n},\mathsf{y})+\mu_j(\mathsf{n},\mathsf{y})-2\mu=(\psi S)^{-1}(\delta_p(\mathsf{n};\kappa)-\delta_p(\mathsf{y};\kappa))$, we therefore obtain that $Z\equiv\delta_p(\mathsf{n};\kappa)-\delta_p(\mathsf{y};\kappa)$ is implicitly defined by the following expression:

$$\psi S(1-2\mu)\frac{\frac{R}{\chi}\frac{\mu(1-\mu)}{6}Z+b\mu(1-\mu)}{\int_{-\kappa}^{\kappa}l_\mu(z)(1-l_\mu(z))}-Z=0$$

$$\Leftrightarrow\mathcal{F}(Z)\equiv\psi S(1-2\mu)\frac{\frac{R}{\chi}\frac{\mu(1-\mu)}{6}Z+b\mu(1-\mu)}{\frac{R}{\chi}\frac{1}{6}Z+b\mu(1-\mu)}-Z=0$$

where the second line comes from the fact that

$$\int_{-\kappa}^{\kappa} l_\mu(z)(1 - l_\mu(z)) = \frac{R}{\chi}\frac{1}{6}Z + b\mu(1-\mu).$$

Notice that (i) since $\psi S < 1$, $Z \in (-1, 1)$, (ii) $\mathcal{F}' < 0$, (iii) $\mathcal{F}(0) > 0$ and (iv) $\mathcal{F}(1) < 0$, we must have $Z \in (0, 1)$, which implies that $\mu_{\text{split}}(l) > \mu$. For κ sufficiently large, $\mu < 1/2$ and expression (40) implies that $\delta_p(\mathsf{n}; \kappa) - \delta_p(\mathsf{y}; \kappa) > 0$.

After verifying that $\mu < \mu_{\text{split}}(l) < 1/2$, the only remaining step is to verify the mixture $l_\tau(\theta)$ on the interval $[\hat{\theta}_\tau(\kappa), \check{\theta}_\tau(\kappa)]$ for each type $\tau \in \{0, b\}$. Observe that the indifference condition for type τ is:

$$R\Big[\lambda_\mu(\theta)\delta_p(\mathsf{y}; \kappa) + (1 - \lambda_\mu(\theta)\delta_p(\mathsf{n}; \kappa)\Big] + \chi(\theta + \tau) = 0, \tag{59}$$

where $\lambda_\mu(\theta) = (1-\mu)l_b(\theta)$ for $[\hat{\theta}_b(\kappa), \check{\theta}_b(\kappa)]$, and $\lambda_\mu(\theta) = 1 - \mu(1 - l_0(\theta))$ for $[\hat{\theta}_b(\kappa), \check{\theta}_b(\kappa)]$. Solving for type $\tau = b$ yields:

$$l_b(\theta) = \frac{R\delta_p(\mathsf{n}; \kappa) + \chi(\theta + b)}{R(1-\mu)[\delta_p(\mathsf{n}; \kappa) - \delta_p(\mathsf{y}; \kappa)]} = \frac{\theta - \hat{\theta}_b(\kappa)}{\check{\theta}_b(\kappa) - \hat{\theta}_b(\kappa)}, \tag{60}$$

as was to be shown. Completing the steps for type $\tau = 0$ is similar. □

We next establish that the equilibrium characterized in the previous lemma is (a) the most-separating symmetric equilibrium for $\mu \geq \frac{1}{2}$, and (b) the unique symmetric equilibrium in which $l_\tau(\theta)$ weakly increases in θ for each $\tau \in \{0, b\}$ when $\mu < \frac{1}{2}$.

Lemma A.3. *For large enough κ, $(l_0^*(\cdot), l_b^*(\cdot))$ is the symmetric equilibrium that maximizes separation among types. Specifically, when κ is sufficiently large:*

1. *when $\mu < 1/2$, there is a unique symmetric equilibrium in which $l_\tau(\theta)$ is weakly increasing in $\theta \in [-\kappa, \kappa]$,*
2. *when $\mu \geq 1/2$, there is a continuum of symmetric equilibria; the one maximizing type separation given by $\{l_0^*, l_b^*\}$, that is*

$$\mathsf{t}_0^*(\theta) = \begin{cases} 0 & \theta < \check{\theta}_0(\kappa) \\ \mathsf{n} \text{ with prob. } 1 & \theta \in \Big[\check{\theta}_0(\kappa), \hat{\theta}_0(\kappa)\Big] \\ \mathsf{n} \text{ with prob. } 0 & \theta \geq \hat{\theta}_0(\kappa) \end{cases}$$

$$\mathsf{t}_b^*(\theta) = \begin{cases} \mathsf{n} \;\; \textit{with prob.} \;\; 1 & \theta < \check{\theta}_b(\kappa) \\ \mathsf{n} \;\; \textit{with prob.} \;\; 0 & \theta \in \left[\check{\theta}_b(\kappa), \hat{\theta}_b(\kappa)\right] \\ \mathsf{n} \;\; \textit{with prob.} \;\; 0 & \theta \geq \hat{\theta}_b(\kappa) \end{cases}$$

.

Proof. We proceed by way of cases, distinguishing between $\mu \geq \frac{1}{2}$ and $\mu < \frac{1}{2}$.

Case 1: $\mu \geq \frac{1}{2}$. For any $l \in \mathcal{L}^*$, observe that $\delta_p(\mathsf{y}; \kappa)$ and $\delta_p(\mathsf{n}; \kappa)$ are fixed, given a strategy profile l. Consider any symmetric equilibrium, l, in which $\delta_p(\mathsf{y}; \kappa) \geq \delta_p(\mathsf{n}; \kappa)$. We can then find $\underline{\theta}(l)$ satisfying:

$$R[(1-\mu)\delta_p(\mathsf{y}; \kappa) + \mu\delta_p(\mathsf{n}; \kappa)] + \chi(\underline{\theta}(l) + b) = 0. \tag{61}$$

Define $\lambda_\mu(\theta; l) \equiv [\mu l_0^*(\theta) + (1-\mu) l_b(\theta)]$. We claim that for all $\theta < \underline{\theta}(l)$, $R[\lambda_\mu(\theta; l)\delta_p(\mathsf{y}; \kappa) + (1 - \lambda_\mu(\theta; l))\delta_p(\mathsf{n}; \kappa)] + \chi[\theta + b] < 0$. Suppose not. This implies that there exists $\theta' < \underline{\theta}(l)$ satisfying

$$R[\lambda_\mu(\theta'; l)\delta_p(\mathsf{y}; \kappa) + (1 - \lambda_\mu(\theta'; l))\delta_p(\mathsf{n}; \kappa)] + \chi[\theta' + b] \geq 0. \tag{62}$$

Since $l_b(\theta') = 0$ is weakly preferred by the mis-aligned type, $l_0(\theta') = 0$ is therefore strictly preferred by the mis-aligned type. Substituting $l_0(\theta') = 0$ into $\lambda_\mu(\theta'; l)$, and comparing expressions (61) and (62), we obtain:

$$\begin{aligned} R[(1-\mu) l_b(\theta')]\delta_p(\mathsf{y}; \kappa) + R[\mu + (1-\mu)(1 - l_b(\theta'))]\delta_p(\mathsf{n}; \kappa) + \chi[\theta' + b] \\ \geq R(1-\mu)\delta_p(\mathsf{y}; \kappa) + R\mu\delta_p(\mathsf{n}; \kappa) + \chi[\underline{\theta}(l) + b]. \end{aligned} \tag{63}$$

Recalling that $\underline{\theta}(l) > \theta'$, and $\delta_p(\mathsf{y}; \kappa) \geq \delta_p(\mathsf{n}; \kappa)$, we obtain a contradiction for any $l_b(\theta') \in [0, 1]$. This implies a mis-aligned type strictly prefers to reject any policy $\theta < \underline{\theta}(l)$. By similar reasoning, in any symmetric equilibrium, l, we can find $\overline{\theta}(l)$ satisfying:

$$R[(1-\mu)\delta_p(\mathsf{y}; \kappa) + \mu\delta_p(\mathsf{n}; \kappa)] + \chi\overline{\theta}(l) = 0, \tag{64}$$

and show that an aligned type strictly prefers to support any policy $\theta > \overline{\theta}(l)$. This further implies that a mis-aligned type strictly prefers to support any policy $\theta > \overline{\theta}(l)$. Thus, we have shown that in any symmetric equilibrium l, for $\tau \in \{0, b\}$: $l_\tau(\theta) = 0$ for $\theta < \underline{\theta}(l)$ and $l_\tau(\theta) = 1$ for $\theta > \overline{\theta}(l)$.

We next observe that $\overline{\theta}(l) = \underline{\theta}(l) + b$, and therefore $\overline{\theta}(l) - \underline{\theta}(l)$ is constant (i.e., b) across all equilibrium strategy profiles and $\theta \sim U$, we can focus without loss of generality on the equilibrium strategy that maximizes type separation inside the interval $[\underline{\theta}(l), \overline{\theta}(l)]$. Recalling our previous lemma that—for κ sufficiently large—there exists an equilibrium l in which: $l_\tau(\theta) = 0$ for $\theta < \underline{\theta}(l)$, $\tau \in \{0, b\}$,

$l_b(\theta) = 1$ and $l_0(\theta) = 0$ for $\theta \in [\underline{\theta}(l), \overline{\theta}(l)]$, and $l_\tau(\theta) = 1$ for $\theta > \overline{\theta}(l)$, $\tau \in \{0, b\}$, it is immediate that this equilibrium maximizes type separation amongst all equilibria in which $\delta_p(\mathsf{y}; \kappa) \geq \delta_p(\mathsf{n}; \kappa)$.

Consider, alternatively, any symmetric equilibrium l in which $\delta_p(\mathsf{y}; \kappa) < \delta_p(\mathsf{n}; \kappa)$. Then, we may implicitly define thresholds $\{\hat{\theta}_\tau(\kappa), \check{\theta}_\tau(\kappa)\}_{\tau \in \{0,b\}}$:

$$D_0(\hat{\theta}'_0(\kappa); l) = R(1-\mu)\delta_p(\mathsf{y}; \kappa) + R\mu\delta_p(\mathsf{n}; \kappa) + \chi\hat{\theta}'_0(\kappa) = 0 \tag{65}$$

$$D_0(\check{\theta}'_0(\kappa); l) = R\delta_p(\mathsf{y}; \kappa) + \chi\check{\theta}'_0(\kappa) = 0 \tag{66}$$

$$D_b(\hat{\theta}'_b(\kappa); l) = R\delta_p(\mathsf{n}; \kappa) + \chi(\hat{\theta}'_b(\kappa) + b) = 0 \tag{67}$$

$$D_b(\check{\theta}'_b(\kappa); l) = R(1-\mu)\delta_p(\mathsf{y}; \kappa) + R\mu\delta_p(\mathsf{n}; \kappa) + \chi(\check{\theta}'_b(\kappa) + b) = 0, \tag{68}$$

We make the following claim, which is also useful when we consider the case $\mu < 1/2$.

Claim 1. *If κ is sufficiently large, then for all μ, in an equilibrium in which $\delta_p(\mathsf{y}; \kappa) < \delta_p(\mathsf{n}; \kappa)$:*

$$l_0^*(\theta) = \begin{cases} 0 & \theta < \hat{\theta}_0(\kappa) \\ \frac{\theta - \hat{\theta}_0(\kappa)}{\check{\theta}_0(\kappa) - \hat{\theta}_0(\kappa)} & \theta \in (\hat{\theta}_0(\kappa), \check{\theta}_0(\kappa)) \\ 1 & \theta > \check{\theta}_0(\kappa) \end{cases}$$

$$l_b^*(\theta) = \begin{cases} 0 & \theta < \hat{\theta}_b(\kappa) \\ \frac{\theta - \hat{\theta}_b(\kappa)}{\check{\theta}_b(\kappa) - \hat{\theta}_b(\kappa)} & \theta \in (\hat{\theta}_b(\kappa), \check{\theta}_b(\kappa)) \\ 1 & \theta > \check{\theta}_b(\kappa) \end{cases}$$

Proof. Observe that $\delta_p(\mathsf{y}; \kappa) < \delta_p(\mathsf{n}; \kappa)$ implies $\hat{\theta}'_b(\kappa) < \check{\theta}'_b(\kappa) < \hat{\theta}'_0(\kappa) < \check{\theta}'_0(\kappa)$. It is immediate that each type $\tau \in \{0, b\}$ of each representative has a strictly dominant strategy to oppose a policy $\theta < \hat{\theta}'_b(\kappa)$, and that each type $\tau \in \{0, b\}$ of each representative has a strictly dominant strategy to support a policy $\theta > \check{\theta}'_0(\kappa)$.

Consider, next, $\theta \in (\check{\theta}'_b(\kappa), \hat{\theta}'_0(\kappa))$. Suppose $l_0(\theta) > 0$; then $l_b(\theta) = 1$. Notice however, that for any such θ:

$$R[1 - \mu + \mu l_0(\theta)]\delta_p(\mathsf{y}; \kappa) + R\mu(1 - l_0(\theta))\delta_p(\mathsf{n}; \kappa) + \chi\theta < R[1-\mu]\delta_p(\mathsf{y}; \kappa) + R\mu\delta_p(\mathsf{n}; \kappa) + \chi\theta < 0,$$

where the first inequality follows from $\delta_p(\mathsf{y}; \kappa) < \delta_p(\mathsf{n}; \kappa)$, and the second inequality follows from $\theta < \hat{\theta}'_0(\kappa)$. Under the conjecture $l_0(\theta) > 0$, we have obtained a contradiction, and conclude that we must have $l_0(\theta) = 0$ for $\theta \in (\check{\theta}'_b(\kappa), \hat{\theta}'_0(\kappa))$. Suppose, second, $l_b(\theta) < 1$; then $l_0(\theta) = 0$. Notice, however, that for any such θ:

$$\begin{aligned}&R[1-\mu]l_b(\theta)\delta_p(\mathsf{y};\kappa)+R[\mu+(1-\mu)(1-l_b(\theta))]\delta_p(\mathsf{n};\kappa)+\chi(\theta+b)\\>&R[1-\mu]\delta_p(\mathsf{y};\kappa)+R\mu\delta_p(\mathsf{n};\kappa)+\chi(\theta+b),\end{aligned}$$

where the first inequality follows from $\delta_p(\mathsf{y};\kappa)<\delta_p(\mathsf{n};\kappa)$, and the second line is strictly positive due to $\theta>\check{\theta}'_b(\kappa)$. Under the conjecture $l_b(\theta)<1$, we have obtained a contradiction, and conclude that we must have $l_b(\theta)=1$ for $\theta\in(\check{\theta}'_b(\kappa),\hat{\theta}'_0(\kappa))$. Thus, in a symmetric equilibrium in which $\delta_p(\mathsf{y};\kappa)<\delta_p(\mathsf{n};\kappa)$, for all $\theta\in(\check{\theta}'_b(\kappa),\hat{\theta}'_0(\kappa))$, we must have $l_0(\theta)=0$ and $l_b(\theta)=1$.

There are two remaining sub-intervals of θ-realizations to consider: $[\hat{\theta}_\tau(\kappa),\check{\theta}_\tau(\kappa)]$ for each $\tau\in\{0,b\}$. Consider, first, $\theta\in(\hat{\theta}'_b(\kappa),\check{\theta}'_b(\kappa))$. Since $\theta<\check{\theta}'_b(\kappa)$, we have that $R\delta_p(\mathsf{n};\kappa)+\chi(\theta+b)<R\delta_p(\mathsf{n};\kappa)+\chi\theta$, so that an aligned type strictly prefers to vote against the policy, i.e., $l_0(\theta)=0$. We argue that, by contrast, we must have $l_b(\theta)\in(0,1)$. To see this, observe that under the conjecture $l_b(\theta)=0$, $\theta>\hat{\theta}'_b(\kappa)$ implies that a mis-aligned type strictly prefers to support the policy; under the conjecture $l_b(\theta)=1$, $\theta<\check{\theta}'_b(\kappa)$ implies that a mis-aligned type strictly prefers to oppose the policy. Thus, we must have $l_b(\theta)\in(0,1)$, i.e., to solve:

$$R[1-\mu]l_b(\theta)\delta_p(\mathsf{y};\kappa)+R[\mu+(1-\mu)(1-l_b(\theta))]\delta_p(\mathsf{n};\kappa)+\chi(\theta+b)=0. \tag{69}$$

We claim that there exists unique $l_b(\theta)$ that solves expression (69) with equality, recalling the dependence of $\delta_p(\mathsf{y};\kappa)$ and $\delta_p(\mathsf{n};\kappa)$ on $l_b(\theta)$. Existence is trivial; to obtain uniqueness, observe that the derivative of the LHS with respect to l_b is:

$$R(1-\mu)(\delta_p(\mathsf{y};\kappa)-\delta_p(\mathsf{n};\kappa))+R(1-\mu)l_b\frac{\partial\delta_p(\mathsf{y};\kappa)}{\partial l_b}+R[\mu+(1-\mu)(1-l_b)]\frac{\partial\delta_p(\mathsf{n};\kappa)}{\partial l_b}. \tag{70}$$

The first term is strictly negative, since $\delta_p(\mathsf{y};\kappa)-\delta_p(\mathsf{n};\kappa)<0$, by conjecture. The partial derivative in the second term is $\psi S\frac{\partial\hat{\mu}(\mathsf{y},\mathsf{y})}{\partial l_b}<0$; and, the partial derivative in the third term is $-\psi S\frac{\partial\hat{\mu}(\mathsf{n},\mathsf{n})}{\partial l_b}<0$. We conclude that there exists a unique $l_b(\theta)$ that solves (69), which is easily verified to take the form $l_b(\theta)=\frac{\theta-\check{\theta}_b(\kappa)}{\hat{\theta}_b(\kappa)-\check{\theta}_b(\kappa)}$. Similar arguments establish that on the remaining interval $[\hat{\theta}'_0(\kappa),\check{\theta}'_0(\kappa)]$ we must have $l_b(\theta)=1$, and that there exists a unique $l_0(\theta)\in(0,1)$ solving $R[1-\mu+\mu l_0(\theta)]\delta_p(\mathsf{y};\kappa)+R\mu(1-l_0(\theta))\delta_p(\mathsf{n};\kappa)+\chi\theta=0$, corresponding to $l_0(\theta)=\frac{\theta-\check{\theta}_0(\kappa)}{\hat{\theta}_0(\kappa)-\check{\theta}_0(\kappa)}$. □

Observe under these strategies, $2\mu_{\text{split}}(l)$ is given by expression (45), which we have noted to be strictly less than 1, i.e., $\mu_{\text{split}}(l)<1/2$. But since $\mu>1/2$, expression (40) highlights that for κ large enough, $\mu_{\text{split}}(l)<1/2$ implies that $\delta_p(\mathsf{y};\kappa)>\delta_p(\mathsf{n};\kappa)$, a contradiction.

Case 2: $\mu<\frac{1}{2}$. The proof that the equilibrium characterized in the previous lemma is the unique symmetric equilibrium in which $\delta_p(\mathsf{n};\kappa)>\delta_p(\mathsf{y};\kappa)$ follows from Claim 1.

Consider, instead, a symmetric equilibrium profile in which $\delta_p(\mathsf{y};\kappa) \geq \delta_p(\mathsf{n};\kappa)$. Suppose, moreover, that this strategy l has the property that for each $\tau \in \{0, b\}$, $l_\tau(\theta)$ weakly increases in $\theta \in [-\kappa, \kappa]$. If $\delta_p(\mathsf{y};\kappa) \geq \delta_p(\mathsf{n};\kappa)$, then $\hat{\theta}'_b(\kappa) \geq \check{\theta}'_b(\kappa)$, $\hat{\theta}_0(\kappa) \geq \check{\theta}'_0(\kappa)$, and $\check{\theta}'_b(\kappa) < \hat{\theta}'_0(\kappa)$. For any $\theta > \hat{\theta}'_b(\kappa)$, a mis-aligned type has a strictly dominant strategy to support the policy. Since this implies $l_b(\theta) = 1$ for $\theta > \hat{\theta}'_0(\kappa)$, for any conjecture $l_0(\theta) \geq 0$, an aligned type strictly prefers to support a policy $\theta > \hat{\theta}'_0(\kappa)$. Thus, $l_0(\theta) = 1$ for $\theta > \hat{\theta}'_0(\kappa)$. Similarly, an aligned type has a strictly dominant strategy to oppose any policy $\theta < \check{\theta}'_0(\kappa)$. Since this implies $l_0(\theta) = 0$ for any $\theta < \check{\theta}'_b(\kappa)$, for any conjecture $l_b(\theta) \geq 0$, a mis-aligned type strictly prefers to oppose a policy $\theta < \check{\theta}'_b(\kappa)$. Thus, we must have $l_b(\theta) = 0$ for all $\theta < \check{\theta}'_b(\kappa)$.

Next, we consider $l_b(\theta)$ for $\theta \in [\check{\theta}'_b(\kappa), \hat{\theta}'_b(\kappa)]$. We claim that $l_b(\theta) > 0$ for some $\theta \in [\check{\theta}_b(\kappa), \hat{\theta}_b(\kappa))$ implies $l_b(\theta') = 1$ for all $\theta' > \theta$. To see this, suppose that there exists $\theta_1, \theta_2 \in [\check{\theta}'_b(\kappa), \hat{\theta}'_b(\kappa)]$ such that $l_b(\theta_1) > 0$ and $l_b(\theta_2) < 1$. Since we are restricting attention to strategies $l_b(\theta)$ that are weakly increasing, we therefore have $0 < l_b(\theta_1) < l_b(\theta_2) < 1$, and moreover individual rationality implies $l_0(\theta_1) = l_b(\theta_2) = 0$. Indifference at each θ implies $R(1-\mu)l_b(\theta_1) + R[1-(1-\mu)l_b(\theta_1)] + \chi(\theta_1 + b) = 0$ and $R(1-\mu)l_b(\theta_2) + R[1-(1-\mu)l_b(\theta_2)] + \chi(\theta_2 + b) = 0$. Re-arranging the first expression yields $l_b(\theta_1) = \frac{\hat{\theta}'_b(\kappa) - \theta_1}{\hat{\theta}'_b(\kappa) - \check{\theta}'_b(\kappa)}$, and rearranging the second yields $l_b(\theta) = \frac{\hat{\theta}'_b(\kappa) - \theta_2}{\hat{\theta}'_b(\kappa) - \check{\theta}'_b(\kappa)}$, and thus $l_b(\theta_2) - l_b(\theta_1) \propto \theta_1 - \theta_2 < 0$, which contradicts the supposition that $l_b(\theta)$ weakly increases in θ. By similar reasoning, we must have that $l_0(\theta) > 0$ for some $\theta \in [\check{\theta}_0(\kappa), \hat{\theta}_0(\kappa))$ implies $l_b(\theta') = 1$ for all $\theta' > \theta$.

We have shown that each type plays a pure strategy for all except (possibly) a measure-zero set of θ, which in turn implies $\mu_{\text{split}}(l) = \frac{1}{2}$. But since $\mu < 1/2 = \mu_{\text{split}}(l)$, expression (40) implies $\delta_p(\mathsf{y};\kappa) < \delta_p(\mathsf{n};\kappa)$, contradicting the premise that $\delta_p(\mathsf{y};\kappa) \geq \delta_p(\mathsf{n};\kappa)$.

To conclude the proof of Proposition 1, we observe that for $\mu \geq 1/2$, under the most separating equilibrium, we can use expressions (38) and (39) i.e., in which $\mu_{\text{split}}(l) = 1/2$, to obtain:

$$\lim_{\kappa\to\infty} \hat{\theta}_0(\kappa) = \hat{\theta}_0 = \frac{RS\psi}{\chi}\left[(2\mu-1)(\mu-1/2) + \frac{1}{2\phi S}\frac{\eta-\alpha}{\eta(1-2\alpha)}\right]$$
$$\lim_{\kappa\to\infty} \check{\theta}_b(\kappa) = \check{\theta}_b = \frac{RS\psi}{\chi}\left[(2\mu-1)(\mu-1/2) + \frac{1}{2\phi S}\frac{\eta-\alpha}{\eta(1-2\alpha)}\right] - b.$$

□

Proof of Corollary 1. By inspection, $\frac{\partial\hat{\theta}_0}{\partial\eta} > 0$ and $\frac{\partial\hat{\theta}_0}{\partial\alpha} < 0$ □

Proof of Corollary 2. By inspection, $\frac{\partial\hat{\theta}_0}{\partial\phi} \propto \alpha - \eta$. □

Proof of Proposition 3. First, suppose that $\mu \geq 1/2$. In this case, constituency voters' first-period payoff (under the proposed selection criterion) can be written as

$$\begin{aligned} W &= \int_{-\kappa}^{\check{\theta}_b} q(\mathsf{n},\mathsf{n})\frac{z}{2\kappa}\,dz + \int_{\check{\theta}_b}^{\hat{\theta}_0}[q(\mathsf{n},\mathsf{n}) + 2(1-\mu)\mu\chi + (1-\mu)^2 2\chi]\frac{z}{2\kappa}\,dz + \int_{\hat{\theta}_0}^{\kappa}[q(\mathsf{n},\mathsf{n}) + 2\chi]\frac{z}{2\kappa}\,dz \\ &\propto \kappa^2 - (1-\mu)\check{\theta}_b^2 - \mu\hat{\theta}_0^2 \\ &= \kappa^2 - (1-\mu)b^2 + \hat{\theta}_0(2(1-\mu)b - \hat{\theta}_0) \end{aligned} \tag{71}$$

where the third line comes from the observation that $\check{\theta}_b = \hat{\theta}_0 - b$. We have:

$$\frac{\partial W}{\partial \eta} \propto ((1-\mu)b - \hat{\theta}_0)\frac{\partial \hat{\theta}_0}{\partial \eta} \tag{72}$$

By inspection, $\frac{\partial \hat{\theta}_0}{\partial \eta} > 0$ and $\frac{\partial \hat{\theta}_0}{\partial \alpha} < 0$. Let $\eta^{**} \equiv \max\{\underline{\eta}, \hat{\eta}\}$, where $\hat{\eta}$ solves $\hat{\theta}_o|\hat{\eta} = (1-\mu)b$. If and only if $\eta \geq \eta^{**}$, welfare strictly decreases in η. Since $\frac{\partial \hat{\theta}_0}{\partial \alpha} < 0 < \frac{\partial \hat{\theta}_0}{\partial \eta}$, $\hat{\eta}$ *increases* in α. □

Suppose, instead, $\mu < 1/2$. we can use expressions (38) and (39), to obtain:

$$\begin{aligned} \lim_{\kappa\to\infty}\hat{\theta}_0(\kappa) = \hat{\theta}_0 &= \frac{RS\psi}{\chi}\left[(1-2\mu)(1/6 + 2\mu_{\text{split}}/3 - \mu) + \frac{1}{2\phi S}\frac{\eta-\alpha}{\eta(1-2\alpha)}\right] \\ \lim_{\kappa\to\infty}\check{\theta}_0(\kappa) = \check{\theta}_0 &= \frac{RS\psi}{\chi}\left[1/6 + 2\mu_{\text{split}}/3 - \mu + \frac{1}{2\phi S}\frac{\eta-\alpha}{\eta(1-2\alpha)}\right] \\ \lim_{\kappa\to\infty}\hat{\theta}_b(\kappa) = \hat{\theta}_b &= \frac{RS\psi}{\chi}\left[-1/6 - 2\mu_{\text{split}}/3 + \mu + \frac{1}{2\phi S}\frac{\eta-\alpha}{\eta(1-2\alpha)}\right] - b \\ \lim_{\kappa\to\infty}\check{\theta}_b(\kappa) = \check{\theta}_b &= \frac{RS\psi}{\chi}\left[(1-2\mu)(1/6 + 2\mu_{\text{split}}/3 - \mu) + \frac{1}{2\phi S}\frac{\eta-\alpha}{\eta(1-2\alpha)}\right] - b. \end{aligned}$$

Then:

$$W \propto \kappa^2 - \mu\big[\check{\theta}_0^2 + \hat{\theta}_0\check{\theta}_0 + \hat{\theta}_0^2\big] - (1-\mu)\big[\check{\theta}_b^2 + \hat{\theta}_b\check{\theta}_b + \hat{\theta}_b^2\big]$$

which implies that:

$$\frac{\partial W}{\partial \eta} \propto -\mu\left[\frac{\partial \check{\theta}_0}{\partial \eta}(2\check{\theta}_0 + \hat{\theta}_0) + \frac{\partial \hat{\theta}_0}{\partial \eta}(2\hat{\theta}_0 + \check{\theta}_0)\right] - (1-\mu)\left[\frac{\partial \check{\theta}_b}{\partial \eta}(2\check{\theta}_b + \hat{\theta}_b) + \frac{\partial \hat{\theta}_b}{\partial \eta}(2\hat{\theta}_b + \check{\theta}_b)\right]. \tag{73}$$

Now:

$$\begin{aligned} \frac{\partial \check{\theta}_0}{\partial \eta} &\propto \frac{\partial \zeta(\alpha,\eta,\phi)}{\partial \eta} + \frac{2}{3}\frac{\partial \mu_{\text{split}}}{\partial \eta} \\ \frac{\partial \hat{\theta}_b}{\partial \eta} &= \frac{\partial \check{\theta}_0}{\partial \eta} - \frac{RS\psi}{\chi}\frac{4}{3}\frac{\partial \mu_{\text{split}}}{\partial \eta} \end{aligned}$$

$$\frac{\partial\check{\theta}_b}{\partial\eta} = \frac{\partial\hat{\theta}_0}{\partial\eta} = \frac{\partial\check{\theta}_0}{\partial\eta} - \frac{RS\psi}{\chi}\frac{4}{3}\mu\frac{\partial\mu_{\text{split}}}{\partial\eta}$$

where it is easy to see that $\frac{\partial\zeta}{\partial\eta} > 0$. After some algebra, we obtain that

$$\begin{aligned}\mu_{\text{split}} &= \frac{1}{2}\frac{2\mu(1-\mu)b + (1-\mu)\mu(\check{\theta}_b - \hat{\theta}_b) + \mu\left(1-\frac{\mu}{3}\right)(\check{\theta}_0 - \hat{\theta}_0)}{2\mu(1-\mu)b + (1-\mu)\frac{1+2\mu}{3}(\check{\theta}_b - \hat{\theta}_b) + \mu\left(1-2\frac{\mu}{3}\right)(\check{\theta}_0 - \hat{\theta}_0)} \\ &= \frac{1}{2}\frac{2\mu(1-\mu)\Gamma(\mu_{\text{split}}) + (1-\mu)^2\mu + \mu^2\left(1-\frac{\mu}{3}\right)}{2\mu(1-\mu)\Gamma(\mu_{\text{split}}) + (1-\mu)^2\frac{1+2\mu}{3} + \mu^2\left(1-2\frac{\mu}{3}\right)}\end{aligned}$$

where

$$\Gamma(\mu_{\text{split}}) \equiv \frac{b\chi}{RS\psi\left(\frac{1}{3} + \frac{4}{3}\mu_{\text{split}} - 2\mu\right)}.$$

Since $\Gamma(\mu_{\text{split}})$ depends on η only through μ_{split}, the implicit function theorem implies that $\frac{\partial\mu_{\text{split}}}{\partial\eta} = 0$ and the rest of proof proceeds as in the case of $\mu \geq 1/2$. □

Proof of Corollary 3. Let $\hat{\theta}_0^C$ denote the threshold $\lim_{\kappa\to\infty}\hat{\theta}_0(\kappa)$ evaluated at $\eta = \frac{\alpha}{2\pi}$, and let $\hat{\theta}_0^O$ denote the threshold $\lim_{\kappa\to\infty}\hat{\theta}_0(\kappa)$ evaluated at $\eta = \frac{\alpha}{2(1-\pi)}$. Expression (71) reveals that closed list PR yields strictly better dyadic representation than open list PR if and only if:

$$\kappa^2 - (1-\mu)b^2 + \hat{\theta}_0^C(2(1-\mu)b - \hat{\theta}_0^C) > \kappa^2 - (1-\mu)b^2 + \hat{\theta}_0^O(2(1-\mu)b - \hat{\theta}_0^O), \tag{74}$$

which is equivalent to $2(1-\mu)b(\hat{\theta}_0^C - \hat{\theta}_0^O) > (\hat{\theta}_0^C)^2 - (\hat{\theta}_0^O)^2$. Noting that $\hat{\theta}_0^C < \hat{\theta}_0^O$, we can express this condition:

$$\hat{\theta}_0^C + \hat{\theta}_0^O > 2(1-\mu)b \iff \frac{RS\psi}{\chi}(1-2\mu)^2 > 2(1-\mu)b, \tag{75}$$

from which the claim follows. □

Corollary 4. *The effect of η on dyadic representation depends on the initial level of η and on the fraction of uncommitted voters, i.e., on α:*

i. when η is small and α is large, more flexibility (i.e., increasing η) improves *dyadic representation;*

ii. when η is large and α is large, more flexibility (i.e., increasing η) worsens *dyadic representation.*

Proof. We have that

$$\begin{aligned}\lim_{\alpha\to1/2}\lim_{\eta\to\underline{\eta}}\frac{\underline{\eta}-\alpha}{\underline{\eta}(1-2\alpha)} &= \lim_{\alpha\to1/2}\frac{-(2\pi-1)}{1-2\alpha} = -\infty \\ \lim_{\alpha\to1/2}\lim_{\eta\to\overline{\eta}}\frac{\overline{\eta}-\alpha}{\overline{\eta}(1-2\alpha)} &= \lim_{\alpha\to1/2}\frac{(2\pi-1)}{1-2\alpha} = \infty\end{aligned}$$

which implies that when partisanship (α) is sufficiently large, the sign of the ratio $\frac{\eta-\alpha}{\underline{\eta}(1-2\alpha)}$ determines the sign of all thresholds. $\square$

Proof of Proposition 2. A closed list is equivalent to $\eta \leq \underline{\eta}$. In this case, the threshold $\xi = \xi_\eta$ is never attained when the Incumbent party obtains a single seat, and there are only two pivotal events. Proceeding as in the proof of Lemma 3, we obtain:

$$\begin{aligned} V_i = V_j =& \frac{1}{2}\left\{\begin{array}{c} (\hat{\mu}_i(\mathsf{t}) + \hat{\mu}_j(\mathsf{t}))S + \mathbb{E}[\theta \leq -b]q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \geq b]q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \leq \theta \leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t}), \hat{\mu}_j(\mathsf{t})) + \mathbb{E}[0 \leq \theta \leq b]q(\mathsf{y},\mathsf{y}) \end{array}\right\} \\ &+\frac{1}{2}\left\{\begin{array}{c} (\hat{\mu}_i(\mathsf{t}) + \mu_O)S + \mathbb{E}[\theta \leq -b]q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \geq b]q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \leq \theta \leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t})) + \mathbb{E}[0 \leq \theta \leq b]Q_{[0,b]}(\mu_O) \end{array}\right\} \end{aligned}$$

and

$$\begin{aligned} V_O =& \frac{1}{2}\left\{\begin{array}{c} (\hat{\mu}_i(\mathsf{t}) + \mu_O)S + \mathbb{E}[\theta \leq -b]q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \geq b]q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \leq \theta \leq 0]Q_{[-b,0]}(\hat{\mu}_i(\mathsf{t})) + \mathbb{E}[0 \leq \theta \leq b]Q_{[0,b]}(\mu_O) \end{array}\right\} \\ &+\frac{1}{2}\left\{\begin{array}{c} 2\mu_O S + \mathbb{E}[\theta \leq -b]q(\mathsf{n},\mathsf{n}) + \mathbb{E}[\theta \geq b]q(\mathsf{y},\mathsf{y}) \\ +\mathbb{E}[-b \leq \theta \leq 0]q(\mathsf{n},\mathsf{n}) + \mathbb{E}[0 \leq \theta \leq b]Q_{[0,b]}(\mu_O, \mu_O) \end{array}\right\} \end{aligned}$$

which implies that $\max\{V_i, V_j\} - V_O \propto \hat{\mu}_i(\mathsf{t}) + \hat{\mu}_j(\mathsf{t}) - 2\mu_O + \mathcal{O}(1/k)$, for any list order and any posteriors $\hat{\mu}_i$ and $\hat{\mu}_j$. Using this result and imposing $\eta = \underline{\eta}$ in our benchmark model yields:

$$\begin{aligned} \hat{\theta}_b^{CL} &= \frac{RS\psi}{\chi}\left[\mu - \frac{1}{2} - \frac{1}{2\phi S}\frac{2\pi - 1}{(1-2\alpha)}\right] - b \\ \check{\theta}_b^{CL} &= \frac{RS\psi}{\chi}\left[2\left(\mu - \frac{1}{2}\right)^2 - \frac{1}{2\phi S}\frac{2\pi - 1}{(1-2\alpha)}\right] - b \\ \hat{\theta}_0^{CL} &= \frac{RS\psi}{\chi}\left[2\left(\mu - \frac{1}{2}\right)^2 - \frac{1}{2\phi S}\frac{2\pi - 1}{(1-2\alpha)}\right] \\ \check{\theta}_0^{CL} &= \frac{RS\psi}{\chi}\left[\frac{1}{2} - \mu - \frac{1}{2\phi S}\frac{2\pi - 1}{(1-2\alpha)}\right] \end{aligned}$$

Suppose $\mu \geq .5$ (the extension to $\mu < .5$ is straightforward). Then legislative cohesion is higher under flexible list than under closed list if and only if $\hat{\theta}_0^{CL} - \hat{\theta}_0^{FL} > 0$. Notice that since cohesion is strictly decreasing in η for $\eta \in [\underline{\eta}, \overline{\eta}]$, we have

$$\lim_{\eta \to \underline{\eta}} \hat{\theta}_0^{CL} - \hat{\theta}_0^{FL} \propto (2\mu - 1)(\mu - 1/2 - (\mu - 1/2)) = 0.$$

Under Single Member Districts, party leaders do not choose a list and each representative voter chooses to reelect her incumbent independently. To ensure comparability with our results under multi-member districts, we assume that each district voter's payoff from a competent representative is $2S$, and that the realization of θ is common to both districts. Letting $p_i(\mathsf{t}_i, \mathsf{t}_j)$ denote representative i's prospect of reelection when her vote is $\mathsf{t}_i \in \{\mathsf{y}, \mathsf{n}\}$, and her co-partisan j's vote is $\mathsf{t}_j \in \{\mathsf{y}, \mathsf{n}\}$, we obtain:

$$
\begin{aligned}
p_i(\mathsf{y}, \mathsf{y}) &= \frac{1}{2} + 2\psi S\big(\hat{\mu}(\mathsf{y}, \mathsf{y}) - \mu_O\big) + \mathcal{O}(1/\kappa) \\
p_i(\mathsf{n}, \mathsf{y}) &= \frac{1}{2} + 2\psi S\big(1 - \mu_O\big) + \mathcal{O}(1/\kappa) \\
p_i(\mathsf{y}, \mathsf{n}) &= \frac{1}{2} + 2\psi S\big(- \mu_O\big) + \mathcal{O}(1/\kappa) \\
p_i(\mathsf{n}, \mathsf{n}) &= \frac{1}{2} + 2\psi S\big(\hat{\mu}(\mathsf{n}, \mathsf{n}) - \mu_O\big) + \mathcal{O}(1/\kappa)
\end{aligned}
$$

and, as a result, $\delta_p(\mathsf{y}) \xrightarrow[\kappa \to \infty]{} 2\psi S\big(\mu - 1\big)$ and $\delta_p(\mathsf{y}) \xrightarrow[\kappa \to \infty]{} 2\psi S\big(- \mu\big)$. As a result,

$$
\begin{aligned}
\hat{\theta}_b^{SMD} &= \frac{RS\psi}{\chi} 2\mu - b \\
\check{\theta}_b^{SMD} &= \frac{RS\psi}{\chi} 2\big[(1-\mu)^2 + \mu^2\big] - b \\
\hat{\theta}_0^{SMD} &= \frac{RS\psi}{\chi} 2\big[(1-\mu)^2 + \mu^2\big] \\
\check{\theta}_0^{SMD} &= \frac{RS\psi}{\chi} 2(1-\mu).
\end{aligned}
$$

When $\mu > 1/2$, $\delta_p(\mathsf{y}) > \delta_p(\mathsf{n})$ and—under the same selection as in the baseline model—we obtain a separating equilibrium with threshold strategies at $\check{\theta}_b$ and $\hat{\theta}_0$. We have that $\min\{\hat{\theta}_\tau, \check{\theta}_\tau\} > -\tau$, which implies that the incentive to cultivate a personal reputation always pushes towards obstructionism.

Comparing $\hat{\theta}_0$ under both systems yields

$$
\hat{\theta}_0^{FL} - \hat{\theta}_0^{SMD} \propto \frac{1}{2\phi S} \frac{\eta - \alpha}{\eta(1-2\alpha)} - (\mu - 1/2) - \mu - (1-\mu)^2.
$$

The RHS strictly increases in η, and and is strictly positive evaluated at $\eta = 1/2$, which implies that there exists $\eta^{SMD} \geq \underline{\eta}$ such that $\hat{\theta}_0^{FL} - \hat{\theta}_0^{SMD} > 0 \Leftrightarrow \eta > \eta^{SMD}$. □

Proof of Proposition 6. The characterization is stated in Lemma A.2, while uniqueness is established in Lemma A.3. The result on first-period payoffs is proven for $\mu \geq 1/2$ and $\mu < 1/2$ in Proposition 3. □

B. Supplemental Appendix: Additional Results on District Magnitude

Our benchmark presentation considers a district represented by two incumbent legislators; in this extension, we consider a setting with n incumbent co-partisan legislators. Each representative is either *aligned* or *misaligned*, with independently drawn probability $\mu = 1/2$. At the election, voters have access to at least n opposition candidates, each of which is aligned or misaligned with independently drawn probability $\mu_O = 1/2$. As in our benchmark, all incumbent representatives observe their own alignment, as well as $\theta \sim U[-\kappa, \kappa]$, the constituency value to the Incumbent party's policy agenda.

We modify the information structure by assuming that whenever $s \in \{1, ...n\}$ representatives vote *nay*, all voters observe the identity of exactly one of these s representatives, each with probability $\frac{1}{s}$. We make this assumption for tractability, but we also view it as realistic: it captures the difficulty that voters have in learning precisely about the activities of all their politicians. In a district represented by more than fifty politicians, it is implausible to expect voters to keep track of the voting behavior of *every* representative: on the other hand, voters may learn of a backbench rebellion via news, and learn of the identity of one of the rebels involved—say a faction leader or prominent politician.

Thus, the timing is:

1. The date-1 constituency value θ, and the alignment of each incumbent $\tau_i \in \{0, b\}$ for $i \in \{1, ..., n\}$ are independently realized. Each representative $i \in \{A, B\}$ observes θ, and her own alignment τ_i.

2. *Date 1 Legislative Interaction*: each representative votes *aye* (y) or *nay* (n). This generates a vote tally $\mathsf{t} = (\mathsf{t}_1, ..., \mathsf{t}_n)$ that is observed by the party leadership.

3. *Voter Learning*: Let $D(\mathsf{t})$ denote the set of representatives that vote *nay* at date 1. If $D(\mathsf{t}) = \emptyset$ all voters observe a common public signal $\zeta_\emptyset$. If $|D(\mathsf{t})| = s > 1$, all voters and the party leadership observe a (common) public signal ζ_i that identifies exactly one of the representatives $i \in D(\mathsf{t})$ that voted *nay*—each representative is selected with probability $\frac{1}{s}$.

4. *List Assignment*: the leadership chooses a ballot order. Voters observe the list, and formulate values $\{V_i\}_{i=1}^n$ and V_O.

5. *Election*: the random variables ξ and σ_J are realized, voters cast their ballots, and seats are allocated. Consistent with the baseline model, committed voters randomize uniformly among

candidates from the party that they most prefer (i.e., a fraction α for the Incumbent party and a fraction α for the Opposition).

6. *Date 2 Legislative Interaction*: the date-2 constituency value θ—as well as the alignment of any newly elected Opposition representatives—are independently realized. Each representative votes *aye* (y) or *nay* (n).

7. Payoffs are collected, and the interaction ends.

Open-List PR. We construct a symmetric equilibrium characterized by thresholds

$$(\check{\theta}^O_\tau(\kappa; n), \hat{\theta}^O_\tau(\kappa; n))_{\tau \in \{0,b\}} \tag{76}$$

such that type τ votes *nay* if $\theta \leq \check{\theta}^O_\tau(\kappa; n)$, randomizes over *aye* and *nay* if $\theta \in (\check{\theta}^O_\tau(\kappa; n), \hat{\theta}^O_\tau(\kappa; n))$, and votes *aye* if $\theta \geq \hat{\theta}^O_\tau(\kappa; n)$. For completeness, we specify that regardless of the vote tally, the leadership randomizes uniformly across all possible ranks.[29]

Under this strategy profile, let $\sigma_\tau(\theta)$ denote the probability that a representative type $\tau \in \{0, b\}$ votes *aye* when the constituency value of the project is θ. We define:

$$Y^O(\theta, l_0, l_b) = [l_\mu(\theta)]^{n-1} \frac{\sum_{k=1}^n \Pr(\xi < \xi(k; \zeta_\emptyset))}{n} + (1 - l_\mu(\theta))^{n-1}) \frac{\sum_{k=2}^n \Pr(\xi < \xi(k; \zeta_j))}{n-1}$$

$$N^O(\theta, l_0, l_b) = \sum_{s=0}^{n-1} \binom{n-1}{s} (1 - l_\mu(\theta))^s (l_\mu(\theta))^{n-1-s} \left[\frac{\Pr(\xi < \xi(1; \zeta_i))}{s+1} + \frac{s}{s+1} \frac{\sum_{k=2}^n \Pr(\xi < \xi(k; \zeta_j))}{n-1} \right]$$

where

1. ζ_i indicates that the representative i is revealed to all voters as having voted *nay*,

2. ζ_j indicates that another representative $j \in D(\mathsf{t}) \setminus \{i\}$ is revealed to all voters as having voted *nay*, $\zeta_\emptyset$ indicates that the public signal reveals that no representative voted *nay*,

3. $l_\mu(\theta)$ is a representative's assessment of the probability that any other representative—whose type is not known to her—votes *yea* at policy value θ:

$$l_\mu(\theta) = \mu l_0(\theta) + (1 - \mu) l_b(\theta), \tag{77}$$

[29] Notice that in an open-list context, the leadership's list ranking has no direct consequence for any representative's prospect of reelection, since that prospect is determined entirely by preference votes. Thus, other ranking strategies by the leadership can be supported as part of an equilibrium, e.g., deterministically assigning representative i to rank i.

4. $\xi(k;\zeta)$ solves

$$\alpha + (1-2\alpha)\left(\frac{1}{2} + \phi(\max\{V_1(\zeta), ..., V_n(\zeta)\} - V_O(\zeta)) - \phi\xi(k;\zeta)\right) = \pi(k;n), \tag{78}$$

   and

5. $\pi(k;n)$ is the threshold vote share such that a party receives k out of n seats when its vote share lies in the interval $[\pi(k;n), \pi(k+1;n))$.

For simplicity, we fix $\pi(k;n) = \frac{k}{n+1}$.

The first line gives the probability of reelection when a representative i votes *aye* at the project value θ. If all the $n-1$ remaining co-partisans vote *aye*, the representative is randomly assigned to ranks 1 through n. If, instead, at least one of the $n-1$ remaining co-partisans votes *nay*, the representative that is identified to voters receives all the preference votes of uncommitted voters; conditional on the party winning a single seat, this representative is guaranteed reelection. Regardless of their final rank in the list, all representatives who vote *aye* share the votes of the committed voters and their priority in seat allocation is effectively randomized uniformly over ranks 2 through n.

Consider the second line, which gives the probability of reelection when a representative i votes *nay* at the project value θ. If $s \in \{1, ..., n-1\}$ representatives vote *nay*, the public signal ζ_i that reveals representative i's dissent to voters is drawn with probability $\frac{1}{s+1}$, in which case she receives uncommitted voters' preference votes and is elected if at least one seat is won. With remaining probability $\frac{s}{s+1}$ her behavior is not revealed to the voters, she only receives votes from the committed voters and again her priority in the party's seat allocation is effectively randomized uniformly over ranks 2 through n.

By arguments similar to our benchmark model, we have that individual rationality requires that, in an equilibrium, $l_0(\theta) > 0$ implies $l_b(\theta) = 1$, and $l_b(\theta) < 1$ implies $l_0(\theta) = 0$. We let $D^O(\theta, \tau, \{l_\tau\}_{\tau \in \{0,b\}})$ denote the relative value to an incumbent type $\tau \in \{0, b\}$ from supporting the policy θ when the strategy of the remaining incumbents is $\{l_b, l_0\}$:

$$R[Y^O(\theta, l_0, l_b) - N^O(\theta, l_0, l_b)] + \chi(\theta + \tau). \tag{79}$$

We define $\{\hat{\theta}^O_\tau(\kappa;n), \check{\theta}^O_\tau(\kappa;n)\}_{\tau \in \{0,b\}}$ implicitly:

$$D^O(\hat{\theta}^O_0(\kappa;n), 0, \{l_\tau\}_{\tau \in \{0,b\}}) = 0 \tag{80}$$

$$D^O(\check{\theta}^O_0(\kappa;n), 0, \{l_\tau\}_{\tau \in \{0,b\}}) = 0 \tag{81}$$

$$D^O(\hat{\theta}_0^O(\kappa;n), b, \{l_\tau\}_{\tau\in\{0,b\}}) = 0 \tag{82}$$
$$D^O(\check{\theta}_0^O(\kappa;n), b, \{l_\tau\}_{\tau\in\{0,b\}}) = 0. \tag{83}$$

For any strategy $\{l_\tau\}_{\tau\in\{0,b\}}$, these thresholds exist if $-R+\kappa\chi > 0$ and $R + (-\kappa+b)\chi < 0$, which we henceforth assume.

By similar arguments to those used in the benchmark setting, we have that:

$$\max\{V_1,...,V_n\} - V_O|\zeta_i = S\left(\frac{\mu\frac{\int_{-\kappa}^{\kappa}(1-l_0(z))dz}{\int_{-\kappa}^{\kappa}(1-l_\mu(z))dz} + (n-1)\mu}{n} - \mu_O\right) + \mathcal{O}(1/\kappa).$$
$$\max\{V_1,...,V_n\} - V_O|\zeta_\emptyset = S\left(\frac{\mu\frac{\int_{-\kappa}^{\kappa}l_0(z)dz}{\int_{-\kappa}^{\kappa}l_\mu(z)dz} + (n-1)\mu}{n} - \mu_O\right) + \mathcal{O}(1/\kappa). \tag{84}$$

We obtain:

$$\hat{\theta}_0^O(\kappa;n) = \frac{R\psi(\Delta(\kappa)+n-1)}{2(1-2\alpha)(n+1)\chi\phi} \tag{85}$$
$$\check{\theta}_0^O(\kappa;n) = \frac{2^{-n}R\psi\left(\Delta(\kappa)+2^n-2\right)}{(1-2\alpha)(n+1)\chi\phi} \tag{86}$$
$$\hat{\theta}_0^O(\kappa;n) = \frac{2^{-n}R\psi\left(\Delta(\kappa)+2^n-2\right)}{(1-2\alpha)(n+1)\chi\phi} - b \tag{87}$$
$$\check{\theta}_0^O(\kappa;n) = \frac{R\psi}{2(1-2\alpha)(n+1)\phi\chi} - b. \tag{88}$$

where

$$\Delta(\kappa) \equiv (2\alpha-1)(n+1)S\phi\left(\frac{1}{n}\frac{\int_{-\kappa}^{\kappa}l_0(z)dz}{\int_{-\kappa}^{\kappa}l_\mu(z)dz} - \frac{1}{n}\frac{\int_{-\kappa}^{\kappa}(1-l_0(z))dz}{\int_{-\kappa}^{\kappa}(1-l_\mu(z))dz}\right). \tag{89}$$

We therefore obtain:

$$\lim_{\kappa\to\infty}\hat{\theta}_0^O(\kappa;n) = \hat{\theta}_0^O(n) = \frac{(n-1)R\psi}{2(1-2\alpha)(n+1)\chi\phi}, \tag{90}$$
$$\lim_{\kappa\to\infty}\check{\theta}_0^O(\kappa;n) = \check{\theta}_0^O(n) = \frac{2^{-n}\left(2^n-2\right)R\psi}{(1-2\alpha)(n+1)\chi\phi}, \tag{91}$$
$$\lim_{\kappa\to\infty}\hat{\theta}_b^O(\kappa;n) = \hat{\theta}_b^O(n) = \frac{2^{-n}\left(2^n-2\right)R\psi}{(1-2\alpha)(n+1)\chi\phi} - b, \tag{92}$$
$$\lim_{\kappa\to\infty}\hat{\theta}_b^O(\kappa;n) = \check{\theta}_b^O(n) = \frac{R\psi}{2(1-2\alpha)(n+1)\chi\phi} - b. \tag{93}$$

It is easily verified that $\hat{\theta}_0(n) > \check{\theta}_0(n) > \hat{\theta}_b(n) > \check{\theta}_b(n)$, implying that for κ large enough, $\hat{\theta}_0^O(\kappa;n) > \check{\theta}_0^O(\kappa;n) > \hat{\theta}_b^O(\kappa;n) > \check{\theta}_b^O(\kappa;n)$. The existence of at least one $\sigma_b(\theta)$ that achieves indifference for a mis-aligned type on the interval $[\check{\theta}_b^O(\kappa;n), \hat{\theta}_b^O(\kappa;n)]$ and at least one $\sigma_0(\theta)$ that achieves indifference for an aligned type on the interval $[\check{\theta}_0^O(\kappa;n), \hat{\theta}_0^O(\kappa;n)]$ follows by continuity. Finally, we have:

1. $\lim_{n\to\infty} \hat{\theta}_0^O(n) = \frac{R\psi}{\chi 2\phi(1-2\alpha)}$
2. $\lim_{n\to\infty} \check{\theta}_0^O(n) = 0.$
3. $\lim_{n\to\infty} \hat{\theta}_b^O(n) = -b.$
4. $\lim_{n\to\infty} \check{\theta}_b^O(n) = -b.$